Reformulating Analysis of Variance (ANOVA) Using Order-Constrained Inference

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INTRODUCTION

- Analysis of variance (ANOVA) is commonly used to analyze data sets with categorical response or predictor variables.
- In an exploratory ANOVA setting, researchers evaluate the main and interaction effects against a "no-effect" Null model without specifying their hypotheses a priori.
- By testing multiple main and interaction effects along with follow-up comparisons inflates Type I error rate.
- Using order-constrained inference, researchers can
  - formulate more flexible and nuanced predictions prior to data collection,
  - test predictions implied by the theory rather than the ANOVA framework,
  - examine multiple predictions jointly as a single model.

Schroeder et al. (2019) Study 4

Investigated the effect of handshakes on cooperation in an economic game.

2 (incentive: cooperative vs. competitive) between-pairs ×
2 (handshake: present vs. absent) between-pairs ×
2 (instruction: present vs. absent) within-pairs

120 pairs of participants were randomly assigned into four conditions. Within each pair of participants, one person received instruction (to shake hands or to go to their seat), while the other did not receive any instruction.

P - Probability of participants choosing the option that increases their gain at the expense of their partner's (defected)

- Cooperative condition

	Handshake	No Handshake
Instruction	$P_{\text{🤝}}$	$P_{\text{✗}}$
No Instruction	$P_{\text{🤝}}$	$P_{\text{✗}}$

- Competitive condition

	Handshake	No Handshake
Instruction	$P_{\text{🤝}}$	$P_{\text{✗}}$
No Instruction	$P_{\text{🤝}}$	$P_{\text{✗}}$

SCHROEDER ET AL. (2019) FINDINGS

We present the **ANOVA hypotheses** and the **order constrained hypotheses** on the same parameters. Our precise and restrictive order-constraints can capture the verbal claims more directly and make them testable/falsifiable.

Main Effect of Handshake

Individuals who shake hands are less likely to defect than individuals who do not shake.

$$\frac{1}{4}(P_{\text{🤝}} + P_{\text{✗}} + P_{\text{🤝}} + P_{\text{✗}}) \neq \frac{1}{4}(P_{\text{🤝}} + P_{\text{✗}} + P_{\text{🤝}} + P_{\text{✗}})$$

Main Effect of Incentives

Cooperatively incentivized participants are less likely to defect than competitively incentivized participants.

$$\frac{1}{4}(P_{\text{🤝}} + P_{\text{✗}} + P_{\text{🤝}} + P_{\text{✗}}) \neq \frac{1}{4}(P_{\text{🤝}} + P_{\text{✗}} + P_{\text{🤝}} + P_{\text{✗}})$$

Main Effect of Instruction

In every incentive and handshake condition, participants who are instructed are equally likely to defect as participants (in that condition) who are uninstructed.

$$\frac{1}{4}(P_{\text{🤝}} + P_{\text{✗}} + P_{\text{🤝}} + P_{\text{✗}}) \neq \frac{1}{4}(P_{\text{✗}} + P_{\text{✗}} + P_{\text{✗}} + P_{\text{✗}})$$

Interaction Effect of Incentive and Handshake

Handshakes are effective throughout and are equally effective for cooperatively and competitively incentivized pairs.

$$\frac{1}{2}(P_{\text{🤝}} - P_{\text{✗}} + P_{\text{✗}} - P_{\text{✗}}) \neq \frac{1}{2}(P_{\text{🤝}} - P_{\text{✗}} + P_{\text{✗}} - P_{\text{✗}})$$

Replace post-hoc followup comparisons by

- In each instruction condition, participants who shake hands under cooperative incentives are less likely to defect than those who do not shake hands (also under cooperative incentives).

$$P_{\text{🤝}} \leq P_{\text{🤝}} \quad \text{and} \quad P_{\text{✗}} \leq P_{\text{✗}}$$

- In each instruction condition, participants who shake hands under competitive incentives are just as likely to defect as those who do not shake hands (also under competitive incentives).

$$P_{\text{🤝}} = P_{\text{🤝}} \quad \text{and} \quad P_{\text{✗}} = P_{\text{✗}}$$

Conjunction (of above order constraints) Model

$$0 \leq P_{\text{🤝}} = P_{\text{🤝}} = P_{\text{🤝}} = P_{\text{✗}} \leq P_{\text{✗}} = P_{\text{✗}} = P_{\text{✗}} = P_{\text{✗}} \leq 1$$

RESULTS

The Bayes factor compares the order-constrained model to a baseline model where all probability parameters are unconstrained.

Models	Bayes Factor
Main effect of handshake	0.27
Main effect of incentives	5
Post-hoc follow-up comparisons	4.4
Interaction effect of incentive and handshake and post hoc follow-up comparisons	78
Conjunction Model	6.92

CONCLUSIONS

Exploratory ANOVA analysis limitations:

- Requires alpha level adjustments to account for multiple comparison.
- Cannot specify substantive alternative hypotheses.
- Restricted to state hypotheses in terms of main and interaction effects instead of more complicated relationships.

Order-constrained inference offers the flexibility to examine nuanced predictions that might not be possible under an ANOVA framework and to test multiple predictions jointly as a single model. Moreover, order-constrained modeling directly tests the scientific predictions themselves, rather than the "no effect" null hypotheses.

References

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