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Probability Updating When Drawing Signals **Without** Replacement

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Conceptual Background

Setup:

You face two decks of cards: one predominantly red, the other predominantly black.

You draw a (red) card.

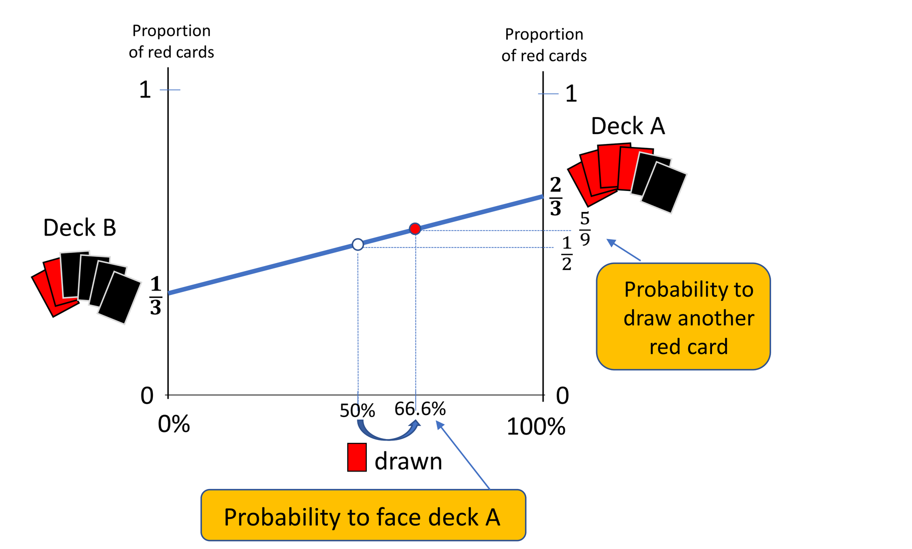
What is the updated probability that you face the predominantly red deck?

or

What is the updated probability to draw another red card?

The standard paradigm in experimental research on probability updating

Drawing **with** replacement



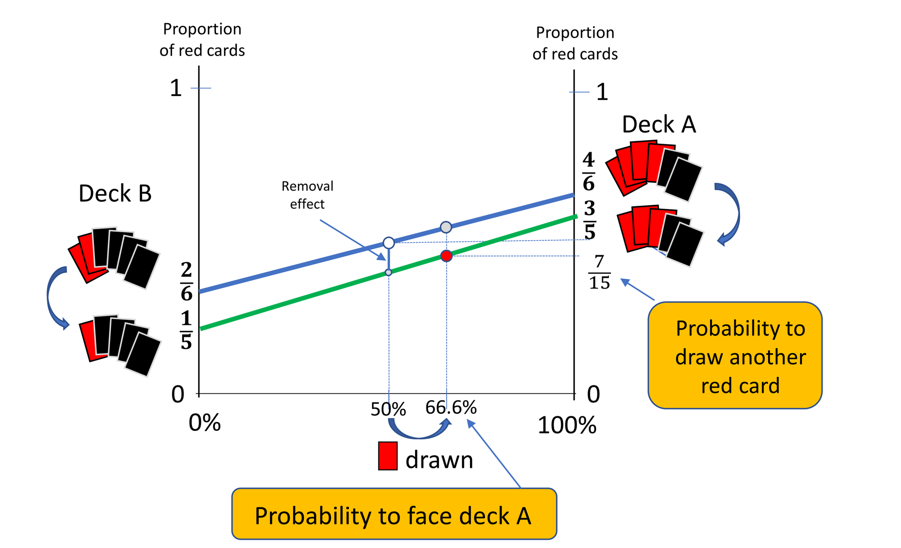
What is different in the two settings?

Next to the „state information effect" there is also a counteracting „removal effect".

Absolute card numbers matter!
The strength of the removal effect depends on the absolute numbers of cards not just their proportions.

A setup mostly neglected in experimental research on probability updating

Drawing **without** replacement



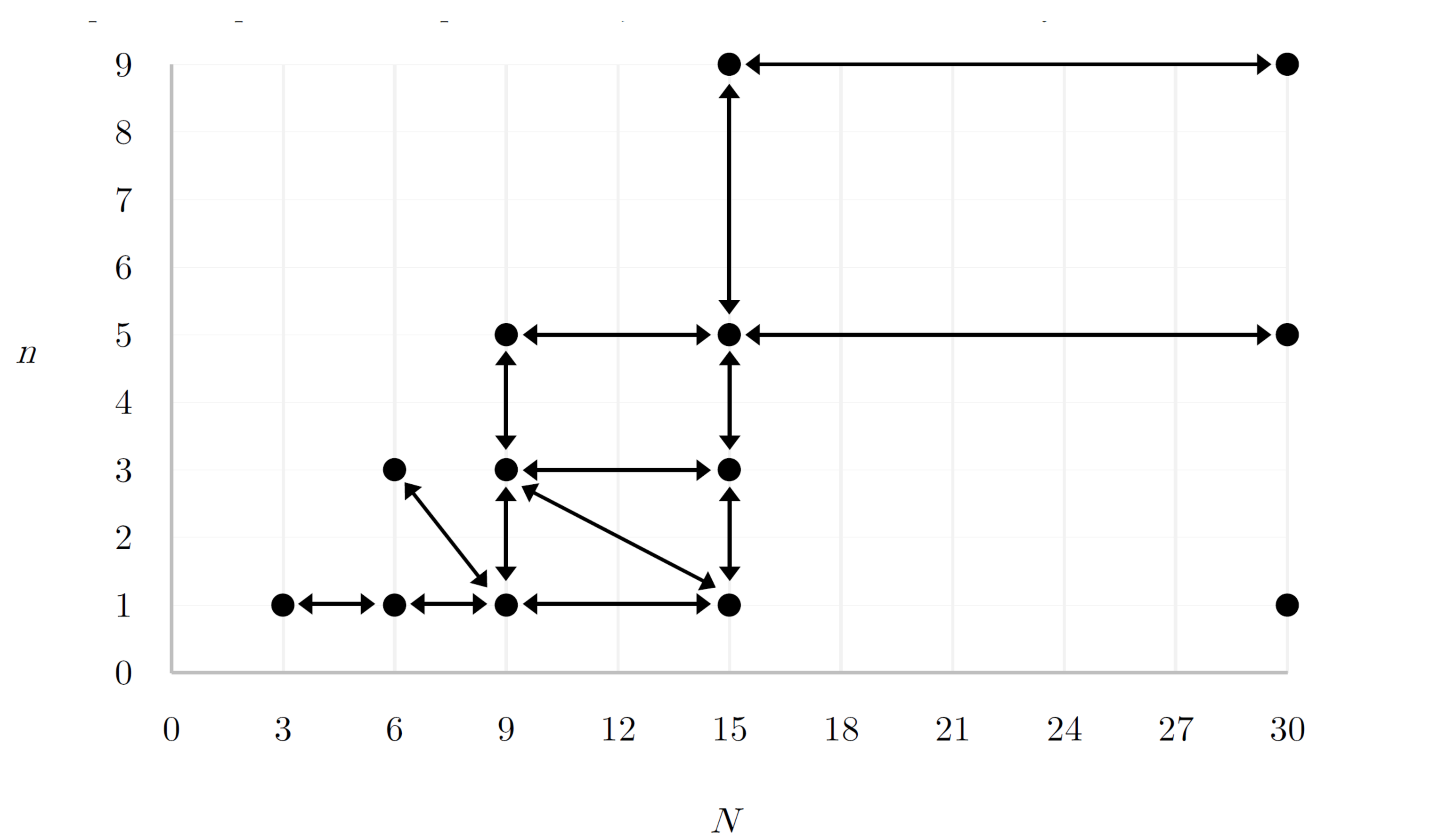
Many real world situations rather resemble a „without replacement" setting:

- High **dividend payments** point to a financially strong firm BUT the money has left the firm.
- High previous sales point to an attractive **product** BUT might also indicate **saturated demand.**
- A high current level of **oil production** suggests generally rich oil deposits BUT might also indicate that the oil field will soon be depleted.
- High past returns of an **investment opportunity** …

Experimental Design

To investigate individuals' probability updating when signals are drawn without replacement, we conducted a preregistered and incentivized lab experiment with 148 students.

Part 1 (live, 5 questions): comparison of outcome probabilities with 50% benchmark → **Part 2** (computer, 14 questions): comparison of outcome probabilities betw. two scenarios → **Part 3** (computer, 13 questions): estimation of explicit outcome probabilities for each scenario

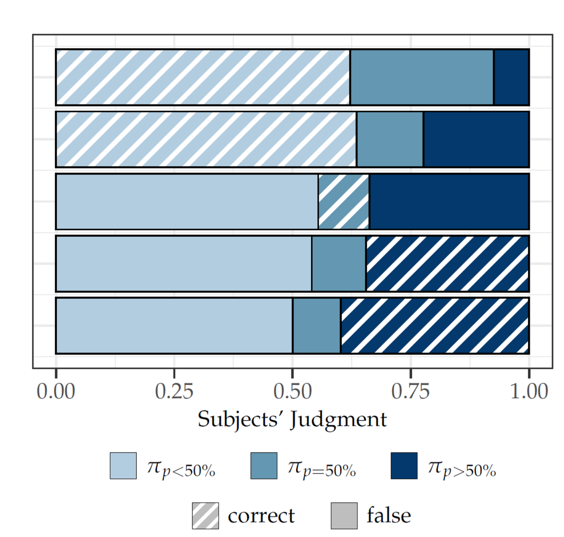


This figure shows the deck size N (horizontal axis) and the number of signals n (vertical axis) of the scenarios considered in the three parts of the experiment. The difference between red and black cards drawn was always 1. Dots with n = 1 mark scenarios which were used in Part 1 of the experiment where subjects had to compare outcome probabilities p with a benchmark probability of 50%. The arrows connect the scenarios for which the subjects had to compare outcome probabilities p in Part 2 of the experiment. In Part 3 of the experiment, subjects had to provide explicit outcome probabilities p for each scenario indicated by a black dot.

Results

Part 1:

N	a	b	Δp_r^*	Δp_s^*	Δp^*	$\pi_{p<50\%}$	$\pi_{p=50\%}$	$\pi_{p>50\%}$
3	1	0	-0.25	0.08	-0.17	0.62	0.30	0.07
6	1	0	-0.10	0.07	-0.03	0.64	0.14	0.22
9	1	0	-0.06	0.06	0.00	0.55	0.11	0.34
15	1	0	-0.04	0.06	0.02	0.54	0.11	0.34
30	1	0	-0.02	0.06	0.04	0.50	0.10	0.40



Subjects observe a red and b black cards drawn without replacement from a deck of size N to judge the posterior probability p to obtain a red card in an additional draw from the chosen deck.

The table provides changes in Bayesian probabilities, that is, the total effect Δp^* as well as its two constituting components removal effect Δp_r^* and state information effect Δp_s^*. The last three columns and the right-hand diagram show the proportion of subjects who believe in p < 50%, p = 50%, and p > 50%. The striped background marks the correct answer for each judgment task.

Part 3:

N	n	a	b	p^*	p_{median}	$p_{median} - p^*$	Δp^*	Δp_r^*	Δp_s^*
3	1	1	0	0.3333	0.3500	0.0167	-0.1667	-0.2500	0.0833
6	1	1	0	0.4667	0.4500	-0.0167	-0.0333	-0.1000	0.0667
6	3	2	1	0.5000	0.4000	-0.1000	0.0000	-0.1667	0.1667
9	1	1	0	0.5000	0.4800	-0.0200	0.0000	-0.0625	0.0625
9	3	2	1	0.5238	0.4600	-0.0638	0.0238	-0.0833	0.1071
9	5	3	2	0.6000	0.4400	-0.1600	0.1000	-0.1250	0.2250
15	1	1	0	0.5238	0.4800	-0.0438	0.0238	-0.0357	0.0595
15	3	2	1	0.5385	0.4900	-0.0485	0.0385	-0.0417	0.0801
15	5	3	2	0.5636	0.4800	-0.0836	0.0636	-0.0500	0.1136
15	9	5	4	0.7143	0.4700	-0.2443	0.2143	-0.0833	0.2976
30	1	1	0	0.5402	0.4900	-0.0502	0.0402	-0.0172	0.0575
30	5	3	2	0.5569	0.4865	-0.0704	0.0569	-0.0200	0.0769
30	9	5	4	0.5844	0.4900	-0.0944	0.0844	-0.0238	0.1082

This table provides information on each of the 13 judgment tasks for which subjects stated explicit outcome probabilities, that is, deck size N , number of cards drawn n, number of red cards drawn a, number of black card drawn b, Bayesian outcome probability p∗, subjects' median outcome probability estimate p_{median}, and the resulting median judgment bias $p_{median} - p^*$.

Key Insights

Part 1:

- A large portion of subjects does not update their beliefs in the correct direction.
- Subjects are not completely insensitive with respect to differences between the five scenarios.
- Subjects' judgment is more in line with the removal effect than with the state information effect.

Part 3:

- Median outcome probability estimates monotonically increase in N for each level of n.
- Subjects tend to systematically underestimate the outcome probability to observe a red card in another draw.

What this paper is about:

Individual biases in probabilistic belief updating have been typically examined in experimental settings where signals are drawn **with** replacement. Motivated by the variety of real world applications, we investigate belief updating in **without** replacement settings.

In such settings, drawing a specific signal has two opposed effects on the likelihood to obtain another signal of the same type in a further draw:

- First, and equivalent to drawing with replacement, the signal provides information on the state of the world resulting in a higher probability to draw the same signal type again.
- Second, the drawn signal is removed reducing the probability to draw the same signal type again.

We find that subjects severely underinfer with respect to the first effect and mildly underestimate the second effect.