Misunderstanding of Place Value Can Explain Logarithmic Compression in Symbolic Number Judgments (145)

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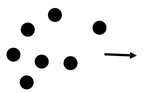
Abstract

Number skills have been linked to several abilities/behaviors, such as health, financial decisions, and risk preferences[1]. We investigated the shape and cause of compression in symbolic number judgments in a ruler task with unfamiliar place value systems.

Theory

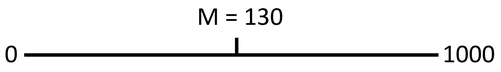
Symbols can be used to indicate number:



e.g.,

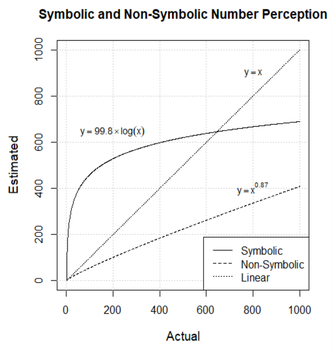
7 (Hindu-Arabic) VII (Roman)

Research shows that children give more space to small numbers on a ruler[2]:



This compression can be explained by assuming a compressive process on the level of a shared mental analogue representation (number module) [3].

Two Problems

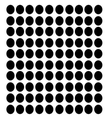


(1) Compression in symbolic number perception tasks[2] looks logarithmic, compression in non-symbolic tasks[4] looks power-function compressed.

(2) Compression in symbolic tasks changes systematically with age, but compression in non-symbolic tasks not.

Solution:
We endorse an alternative explanation[5]:
Children's estimates are shaped by their lack of understanding of the place value system of symbolic numbers.

How would that work? Place value contains:
Compression Repeated Division/Multiplication



→ 100

If children do not know that there are many more numbers between 10-100 than between 0-10, they would assume that there are equally as many numbers in each of these segments (i.e., they would put 10 in the middle of a 0-100 ruler)[5].

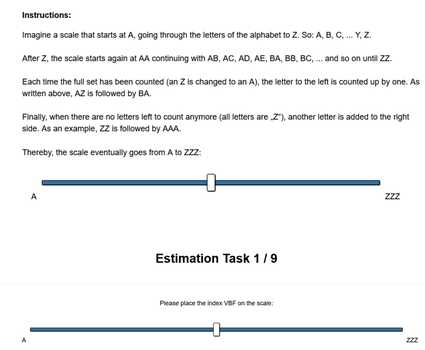
References:
[1] Garcia-Retamero, R., Sobkow, A., Petrova, D., Garrido, D., & Traczyk, J. (2019). Numeracy and Risk Literacy: What Have We Learned so Far? *The Spanish Journal of Psychology*, *22*.
[2] Siegler, R. S., & Opfer, J. E. (2003). The development of numerical estimation: Evidence for multiple representations of numerical quantity. *Psychological Science*, *14*(3), 237–250.
[3] Dehaene, S. (1992). Varieties of numerical abilities. *Cognition*, *44*(1), 1–42.
[4] Indow, T., & Ida, M. (1977). Scaling of dot numerosity. *Perception & Psychophysics*, *22*(3), 265–276.
[5] Moeller, K., Pixner, S., Kaufmann, L., & Nuerk, H.-C. (2009). Children's early mental number line: Logarithmic or decomposed linear? *Journal of Experimental Child Psychology*, *103*(4), 503–515.
[6] Scheibehenne, B. (2019). The psychophysics of number integration: Evidence from the lab and from the field. *Decision, 6*(1), 61–76.

Task (Methods)

How to test this (misunderstanding of place value)?
We give adults an unfamiliar place value system on which they cannot do arithmetic.

letters instead of numerals

Base-26 & Base-5



Nine indices randomly drawn from relatively equally spaced bins.

N = 188, participants on prolific (58% women, Mage = 36.0 years, SD = 14.6)

Theoretical Predictions

(A) Power function shape if compression happens on shared mental analogue representation.
(B) Linear function if adults learned to adopt a more correct linear representation.
(C) Logarithmic shape if the place value system leads to systematically biased estimates.
(D) Sequential linear shape if adults are under the misconception that there is linear growth with each additional figure.

Analysis

Model comparison (cross validation):

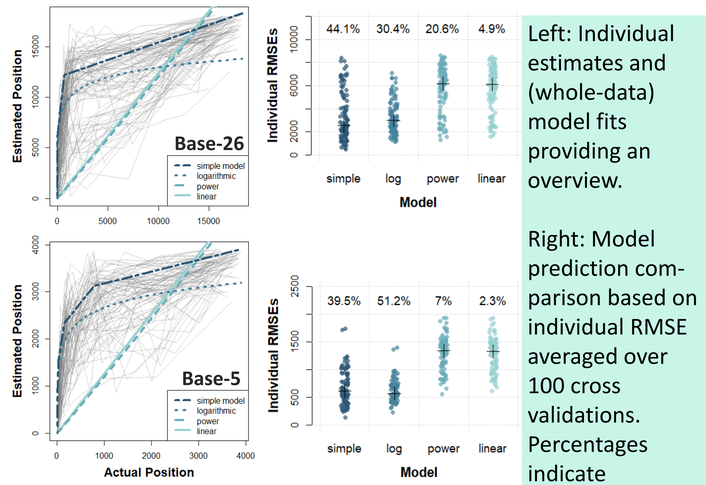
Power: $y = x^b$
Linear: $y = a \cdot x$
Logarithmic: $y = a \cdot log(x)$

Random slope, fixed intercept

Simple Model (sequential linear). No free parameters.

Results

The **logarithmic model** and the **simple model** make the best predictions.



Left: Individual estimates and (whole-data) model fits providing an overview.

Right: Model prediction comparison based on individual RMSE averaged over 100 cross validations. Percentages indicate proportion of participants best predicted by the model.

The logarithmic model and simple model perform better than the other two models based on binomial tests (all $BF > 5'000$, $p < .001$).

Discussion

We found evidence for logarithmic compression in symbolic number judgments being caused by the place value system of symbolic numbers.

This provides a simple and parsimonious explanation for children's logarithmic-looking estimates (and potentially the developmental shift).

Future research can reveal whether place value understanding might be a predictor for other abilities/behaviour such as underestimation biases (e.g., undersum bias)[6] or risk aversion.

Zoom Q&A: https://kit-lecture.zoom.us/j/65047856847

Preprint: https://psyarxiv.com/9f3z2/

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