Approaching the Hot Hand with a Cold Head

Máté Cs. Sándoré [1] Barna Bakó [1]

[1]Department of Economics, Corvinus University of Budapest

CORVINUS UNIVERSITY of BUDAPEST

The SatoshiDice game

SatoshiDice is a simple dice-type game that used to be played only using BitCoin transactions. The game plays as follows:

- Select a risk level from a preexisting list of win probabilities / multipliers
- Initiate a bet with a bet by transferring to the address of the chosen game
- A return transaction of the prize (win) or a infinitesimal amount (loss) is sent back by the platform, based on the (provably) random outcome

The webpage features 27 choices for risk ranging from the winning probability of $.002\%$ (multiplier of $\times 65536$) up to 98% (multiplier of $\times 1.02$), with more options offered on the low end.

The dataset

The BitCoin blockchain provides a public ledger of transactions. From this we have extracted close to a million bets placed at satoshidice.com, from its peak popularity at 2012 and '13. The data contains information about:

- **Users** Estimated anonymous contraction of addresses using Kondor et al. (2014).
- **Risk** Defines the multiplier
- **Bet placed**
- **Outcome** Generated using an algorithm matching incoming and outgoing transactions from the game.

	Start date	No. of bets	No. of users	Total (BTC)	Median bet (BTC)	USD/BTC
A	12-05-02	119,713	1012	47,639	0.03	5
B	12-09-17	130,810	2122	85,343	0.05	12
C	12-12-17	254,617	3419	416,153	0.03	13
D	13-05-04	350,836	3509	102,451	0.02	111
E	13-09-11	91,790	1410	64,468	0.03	123

Table 1: Summary statistics of the 5×21 day samples used. We have chosen periods of relatively low price volatility. The exchange rate is an average over the sample period.

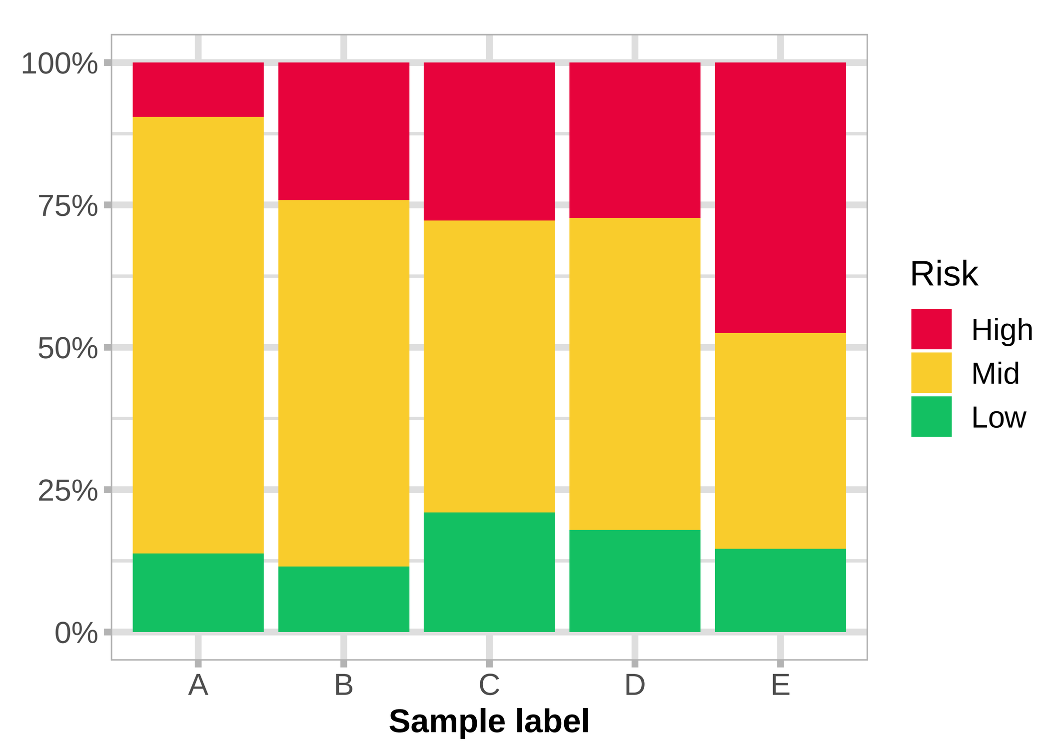

Figure 1: Distribution of risk taken in the sample populations (High: $P_{win} < 33.3\%$, Low: $P_{win} > 66.6\%$).

Hot hand and gambler's fallacy

1. **Gamblers fallacy** is a belief in negative autocorrelation of a non-autocorrelated sequence of outcomes. Generally referred to as a pattern in gambling where losers think their "luck must turn".
2. **Hot hand** is a belief in a positive autocorrelation of a non-autocorrelated sequence of outcomes. Generally referred to as a pattern in gambling where winners think they are on a "hot streak" and will keep on winning.

Croson and Sundali (2005) have processed video recordings of 139 roulette players placing 24,131 bets. They have found that players show signs of both biases.

Xu and Harvey (2014) have examined a dataset of online sports betting consisting of 565,915 bets of 776 players. **They have stated that the above are not fallacies, but observable effects that are caused by adjustments made by the players after positive/negative outcomes.** Winners chose safer bets, thus "creating" a hot hand. As evidence, they pointed at the increasing (decreasing) observed winning probability as winning (losing) streaks grew longer.

Observed winning probability of stationary agents vs. the data

Let us consider a game where the risk (P_{win}) is randomly chosen from a cont. scale between 0% and 100% which the players stick to. The risk density of users experiencing a winning streak of length n is $\Phi_w^n(P_{win}) = (n+1)P_{win}^n$. The expected winning probability of players in the streak of length n is:

$$\chi_w^n = \int_0^1 P_{win}\Phi_w^n(P_{win})dP_{win} = \int_0^1 (n+1)P_{win}^{n+1}dP_{win} = \frac{n+1}{n+2}$$

As the above shows, **even stationary agents show increased observed winning probability as streaks get longer, without adjusting their risk**. We have checked the emergence of this effect using simulations on different distributions of risk choice as well as on our data. As Figure 2. shows, the convergence of observed winning probability is present both in our dataset and the simulation data. We argue that **the convergence is not the result of a change in player's behavior**, but a mathematical consequence.

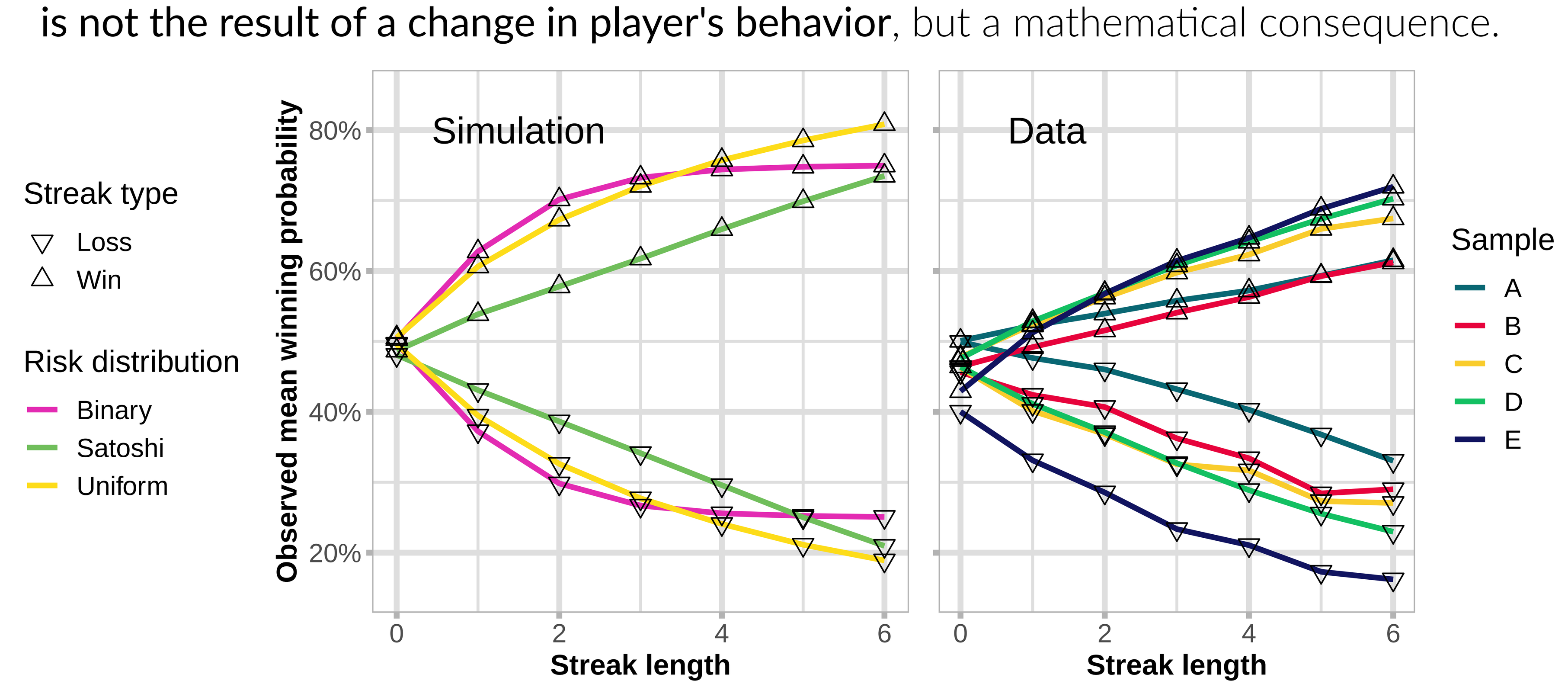

Figure 2: Observed winning probability vs streak length for winning and losing streaks for simulations of stationary agents (left) and our dataset (right).

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Changes in player behavior vs. streaks

Examining the risk strategies in consecutive games we found that players do not change their winning probability in the majority of cases. Streak length however does have a positive (negative) effect on changing strategies over losing (winning) streaks.

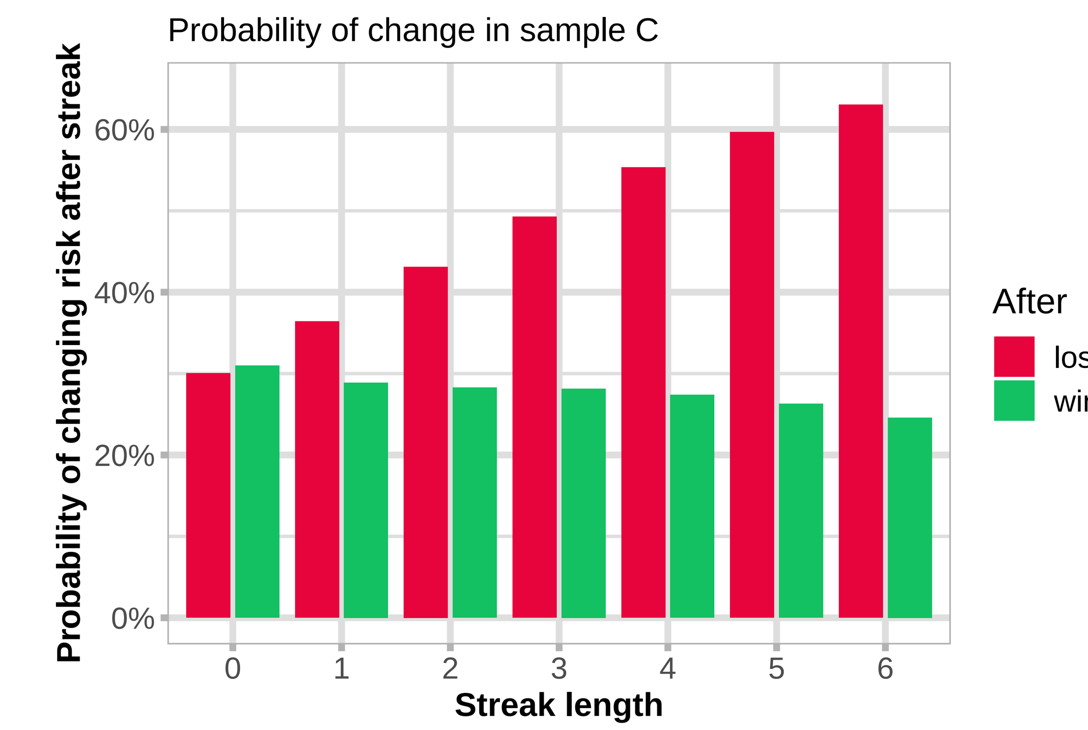

Figure 3: The probability of change vs length of win/loss streaks in sample C (right). The tendencies shown are visible in all samples.

Figure 4. demonstrates the direction and amount of change in winning probability. When change does occur it is consistently negative for winning streaks. Importantly the opposite can be seen on the losing side. These effects clearly contradict Xu and Harvey (2014).

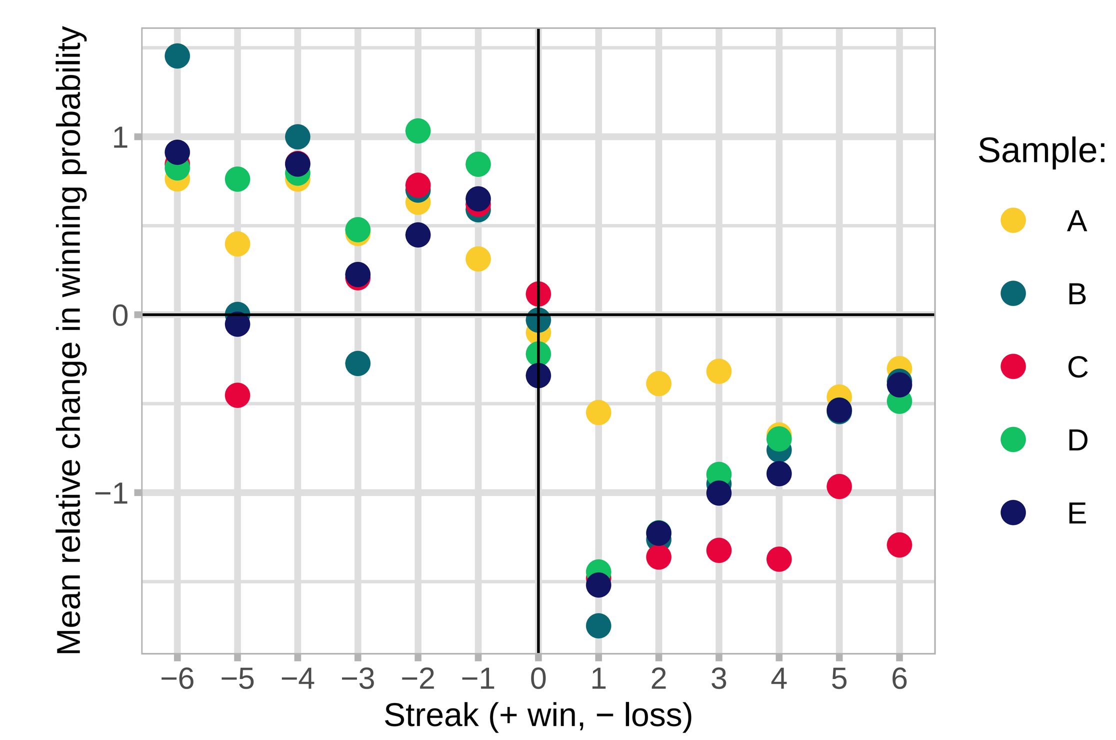

Figure 4: Median of relative change in winning probability over the samples (left) here a negative value means an increase in risk.

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