ZOOM SESSION 2 3:15-4:00 LINK

How Big is (Sample) Space? Judgement and Decision Making with Unknown States and Outcomes

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Link for zoom session:

- Many real-world decisions must be made when we do not know all of the possible relevant states beforehand, i.e., the "sample space" (Ω).
- Standard probability theory takes a complete Ω for granted.
- There is scant psychological research on how people construct Ω. Field biologists have been constructing Ω is for decades, so we've borrowed ideas from them and from biostats to generate questions, hypotheses, and models.

We report three experiments investigating judgments about the nature of Ω.

Study 1 tests whether people use reasonable heuristics for estimating the size of Ω in a simple sample-resample situation.
Studies 2 and 3 show that the biologists' intuition that a larger number of unique events in a sample implies a larger Ω also applies to many laypeople, but this can be overridden by prior beliefs about Ω.
Study 3 demonstrates that greater *qualitative* diversity in a sample also magnifies assessments of the size of

Study 1: Sample-Resample Estimation

The experimental task requested participants to make an estimate given a scenario such as this one:

"A biologist is trying to estimate the number of fish species in a lake. The species are evenly scattered throughout the lake. She drags a large net through the length of the lake and catches 100 different species of fish. She tags them and releases them back into the lake. Shortly thereafter, she drags the net through the lake a second time and again catches 100 different species of fish. She finds that 90 of these are new species she had not found in the first catch. What should be her estimate of the total number of fish species in the lake?"

2x2x2 between-subjects design:

- Number of fish species vs number of carp
- Relative frequencies vs percentages
- Info. about newly captured vs recaptured in the second catch

Two response formats (in two independent samples):

1. Multiple-choice (23 alternatives)
2. Free text-entry

Target prescriptive population estimate: The Lincoln-Petersen estimate. Can laypersons produce this? Under what conditions?

Lincoln-Petersen Estimate

we have K_j distinct species captured on the j^{th} sampling occasion out of a total sample size $n_{j\cdot}$, with π_j being the proportion of the total number of species that would be expected to be retrieved on this occasion. Denoting the total number of species existing on this occasion by κ_j, if we have an estimate of π_j then we may estimate κ_j by

$$\hat{\kappa}_j = K_j / \hat{\pi}_j .$$

The classic Lincoln-Petersen estimate of π_j exploits the capture-recapture process, under suitable assumptions. Given a sample whose species were marked and replaced into the population, the proportion of marked species in the first sample turning up in the next sample, gives an estimate of π_2, so

$$\hat{\kappa}_2 = K_2 / \hat{\pi}_2 = K_2 K_1 / M_2 .$$

Hypothesis 1: Recapture information will be more likely than newly-captured information to yield a Lincoln-Petersen estimate.
Hypothesis 2: Estimating a homogeneous population (carp) will be more likely than estimating a heterogeneous population (fish species) to yield a L-P estimate.

Results

The next table shows a clear effect for species vs carp, supporting Hypothesis 2. The L-P estimate is 1000, and people got it or got close (900) more often in the carp population condition.

estimate	species		carp pop.		total	
110	22	11.9%	14	6.7%	36	9.2%
190	87	47.0%	62	29.8%	149	37.9%
900	24	13.0%	42	20.2%	66	16.8%
1000	33	17.8%	62	29.8%	95	24.2%
other	19	10.3%	28	13.5%	47	12.0%
total	185		208		393	

The next table shows an effect for multiple-choice vs text entry format. People were more likely to return 900 in the multiple-choice format and more likely to return an "other" estimate for text entry.

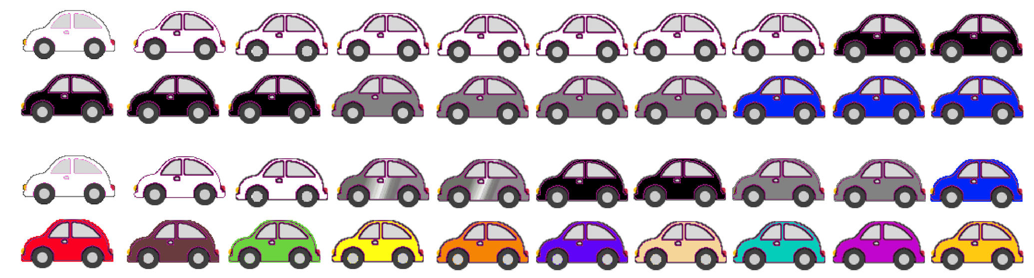
estimate	text entry		mult choice		total	
110	3	1.8%	13	8.3%	16	4.9%
190	36	21.4%	38	24.4%	74	22.8%
900	8	4.8%	29	18.6%	37	11.4%
1000	51	30.4%	55	35.3%	106	32.7%
other	70	41.7%	21	13.5%	91	28.1%
total	168		156		324	

Study 2: Cardinality of Ω, Part I

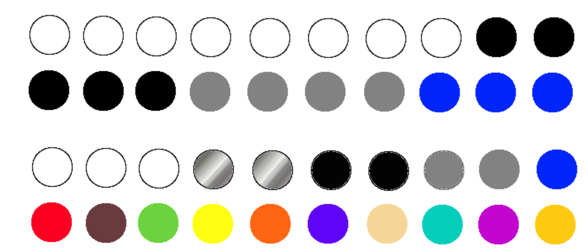
This study was intended to find out whether people have the "biologist's intuition" that the greater the variety of outcomes observed in a sample, the bigger the space. We investigated this for two sample spaces: A bag of marbles of unknown colors, and a city containing automobiles of unknown colors.

2x2-between design:

- Automobiles vs marbles
- A 4-unique-colors sample presented first and a 15-colors sample presented second, or the reverse.



'As we observe 100 more automobiles going through this intersection, how many of them will be colors that are different from the colors we've seen so far?'



'As we draw 100 more marbles from the bag, how many of them will be colors that are different from the colors we've seen so far?'

What did we find?

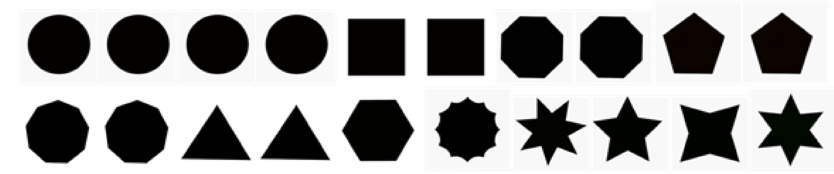
- In the marble conditions, we expected the participants to give "biologist's intuition" responses, i.e., to expect more new colors to be seen when they'd seen 15 colors in their sample than when they'd seen 4 colors in the other sample. About 62% of the participants gave "biologist's intuition" responses.
- In the automobile conditions, we expected the reverse of the "biologist's intuition" response, because participants would believe the number of automobile colors is fairly small. About 63% did the reverse– they expected to see fewer new colors if the sample had 15 colors than if it had just 4 colors.
- L-P estimators were more likely than non-L-P estimators to show the biologist-response in the marbles condition but not more likely in the automobiles condition.

	autos	autos		marbles	marbles	
Estimators	Non-biol.	Biol. intuit.	odds	Non-biol.	Biol. intuit.	odds
non-L-P	189	105	0.556	59	60	1.017
L-P	125	77	0.616	26	76	2.923
odds-ratios			1.109			2.874
total	314	182	0.580	85	136	1.600

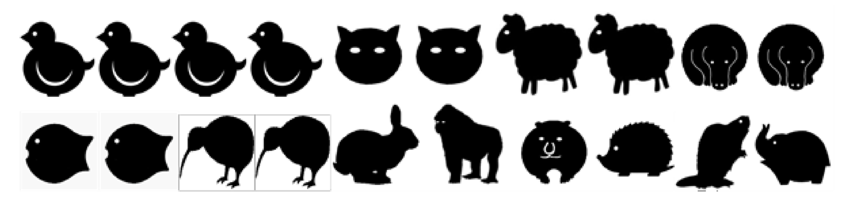
Study 3: Cardinality of Ω, Part II

This study was intended to find out whether *qualitative* diversity in a sample influences beliefs about the size of the sample space. Qualitative diversity was manipulated by using one sample space (regular polygons) that most participants would consider less diverse than another (animals). Experimental stimuli from both sample spaces were drinks coasters.

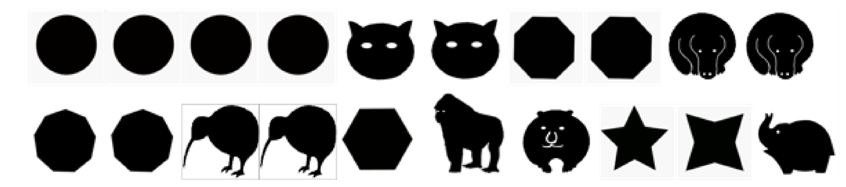
Polygons:



Animals:



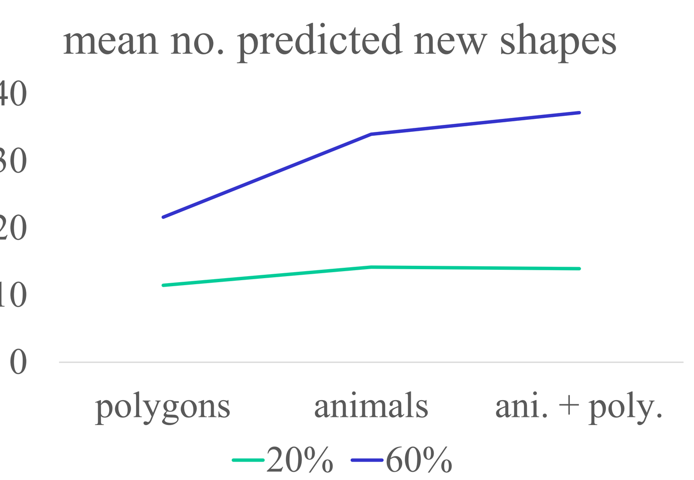
Polygons + Animals



Experimental design: 3 (types of shapes) x 2 (10 vs 20 sample size) x 2 (20% vs 60% being unique shapes)

What did we find?

Participants were asked how many new shapes of coasters expected for a sample of 100 more of them from a city's shops. There were more "biologists' intuition" responses for the animals and animals + polygons shapes than for the polygons shapes, and larger estimates in those two conditions.



Bio.intuit.?	polygons	animals	anim. + poly.
yes	54.2%	69.4%	85.6%
opposite	39.3%	23.1%	12.4%
other	6.5%	7.4%	2.1%

Conclusions

- Laypeople generally use reasonable heuristics in estimating the size of Ω given capture-recapture information under ideal conditions.
- Greater quantitative and qualitative diversity in samples from an unfamiliar Ω increase estimates of its size, but this can be overridden by prior beliefs about Ω.