The Dual Accumulator Model of Strategic Deliberation and Decision Making

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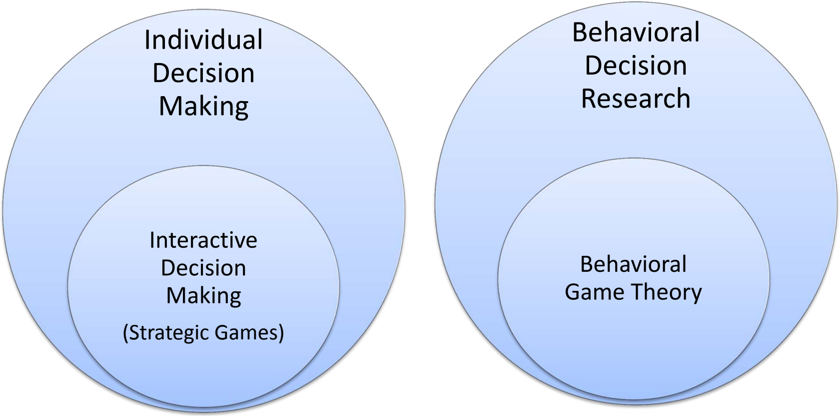
Behavioral Game Theory



Three Components of Behavioral Game Theory

"Virtually all [experimental] results. . . can be accommodated by including behavioral components–

- *social utility,*
- *limited iterated reasoning, and*
- *learning*

–into analytical theory"

– Colin Camerer

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Should We Care About Cognitive Processes?

Much of behavioral game theory has grown out of relaxing unrealistic assumptions behind Nash Equilibrium:

- Level-k reasoning and cognitive hierarchy theory relax assumption of accurate beliefs but still rely on best responding
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Describing realistic cognitive processes underlying strategic deliberation may allow us to

- make better behavioral predictions about strategic choice
- make predictions about correlations between strategic choice and response time
- make predictions about attention during deliberation

What Are the Underlying Cognitive Processes?

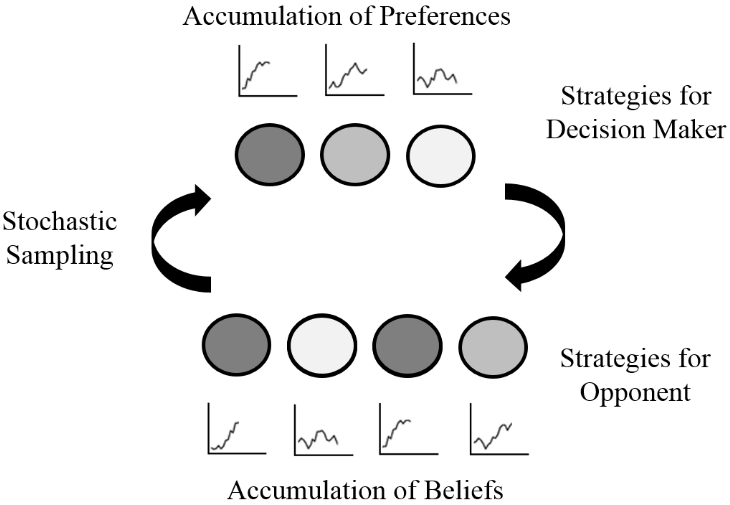
We propose a bidirectional preference accumulation model to describe strategic deliberation

- Stochastic sampling (with fixed probabilities) and dynamic accumulation are cognitive processes that underlie a good behavioral model of risky choice (decision field theory)
  also other forms of preferential choice (i.e., multi-attribute or decentralized)
- We introduce bidirectional feedback (and dynamically changing probabilities) to apply these cognitive processes to strategic choice

Heuristic Implementation

- Sampling:
  1. Consider some strategy s_j my opponent might choose. "What if my opponent chose s_j? How would my strategies do?"
  2. Then consider a strategy s_i that catches my eye / looks good for me to choose. "What if I chose s_i? How would that affect my opponent?"
     (Consideration of a strategy is random, but influenced by how good it currently seems and how salient it is. Influence scaled by a stochastic sampling parameter λ)
- Accumulation: Repeat a finite number of times (T), building up an overall sense of how good each strategy feels, and then pick what feels best.

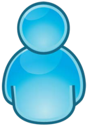
Bidirectional Accumulation

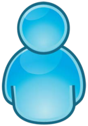


Behavioral Patterns Reflecting Limited Iterated Reasoning

- Stochastic choice
- Failures of unraveling
- Payoff sensitivity
- Risk-reward tradeoffs in coordination games
- Salience effects

Traveler's Dilemma





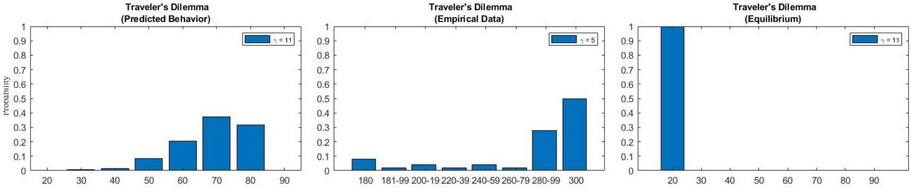
$s_1 \in \{20, 30, \ldots, 90\}$ $s_2 \in \{20, 30, \ldots, 90\}$

Both players get $\min\{s_1, s_2\} \pm \gamma$, where γ is a small reward or penalty given to the player with the lower claim and taken from the player with the higher claim

Nash Equilibrium Prediction
Everyone always claims 20

Real Behavioral Pattern
Higher claims as γ decreases
Claims approach upper bound
(Capra et al., 1999; Goeree & Holt, 2001)

Traveler's Dilemma: Predictions vs Data



- Predicted behavior with $\lambda = .01$ and $T = 10$
- Empirical data reported by Goeree and Holt (2001)

Kreps' Game

P2

		Left	Middle	Non-Nash	Right
P1	Top	70 100	69 60	66 62	10 64
	Bottom	10 60	40 62	66 66	68 70

Nash Equilibrium Prediction

{Top, Left} or {Bottom, Right}

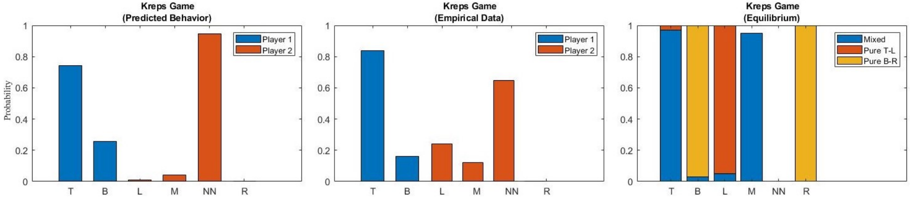
Real Behavioral Pattern

Top and Non-Nash are modal

(Evidence of risk-reward tradeoff)

(Goeree & Holt, 2001)

Kreps' Game: Predictions vs Data



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Strategies with Salient Labels

Choose one of four boxes:

A	B	A	A

to play a

- simple coordination game
- hide-and-seek game
- discoordination game

Nash Equilibrium Prediction

Labels are irrelevant

Locations are irrelevant

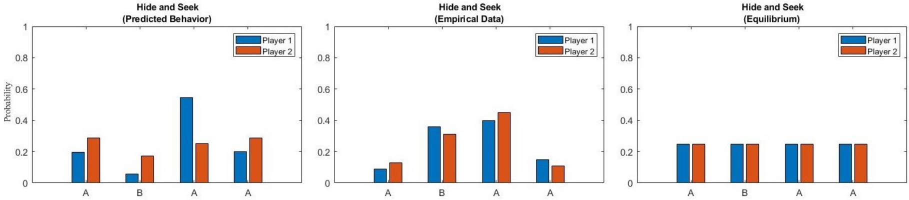
Real Behavioral Pattern

Choose B for coordination

Middle-A modal in hide-and-seek

(Rubinstein et al., 1997)

Hide and Seek: Predictions vs Data



- Predicted behavior with $\lambda = .01$ and $T = 10$ and $\sigma_1 = \sigma_2 = [50, 100, 0, 50]$
- Empirical data reported by Rubinstein et al. (1997)

Theorem

The dual accumulator model predicts that strictly dominated strategies will never be chosen.

Existing Behavioral Game Theory Models

- Level-k reasoning / cog-hierarchy: do k steps of best responding (assuming level 0 is uniformly random or most salient strategy)
    - Accounts for failures of unraveling
    - Accounts for risk-reward tradeoffs
    - Accounts for some, not all salience effects
    - Fails to account for many instances of payoff sensitivity
    - Heterogeneity, but no intrinsic variability
- Logit quantal response equilibrium: a noisy best response to (accurate) expected play of the other player
    - Accounts for payoff sensitivity
    - Accounts for risk-reward tradeoffs
    - Accounts for some, not all failures of unraveling
    - Cannot account for salience effects
    - Stochastic, but too tolerant of dominated strategies

Model Comparison

Fit models to Stahl and Wilson (1995) data

How well can we account for the strategy choices of 48 subjects who each played a set of 12 3x3 symmetric games once without feedback?

Model	MSE Full Sample	MSE Out-of-sample
Dual Accumulator	0.1434	0.1775
Level-k	0.1871	0.2210
Level-k with noise	0.1653	0.2000
Poisson Cognitive Hierarchy	0.1921	0.2068
Empirical Cognitive Hierarchy	0.1944	0.2049
Logit Quantal Response Equilibrium	0.1971	0.2174
Noisy Introspection	0.1104	0.2300

Shared Ingredients

- Level-k reasoning – finite steps of strategic deliberation
- Logit equilibrium – stochastic choice
- Decision field theory – sampling and accumulation

Conclusion

Our model makes good behavioral predictions

Same cognitive mechanisms at play in strategic and non-strategic choice

Formal Structure

Let A_j be the activation for strategy s_j (initially 0)
Let σ_j be the salience of strategy s_j
Let p_j be the probability of considering s_j
Let u_{ij} be the utility i gets from choosing s_i when j chooses s_j

$$p_j = \frac{e^{\lambda(A_j+\sigma_j)}}{\sum_k e^{\lambda(A_k+\sigma_k)}} \qquad A_i \mapsto A_i + u_{ij}$$

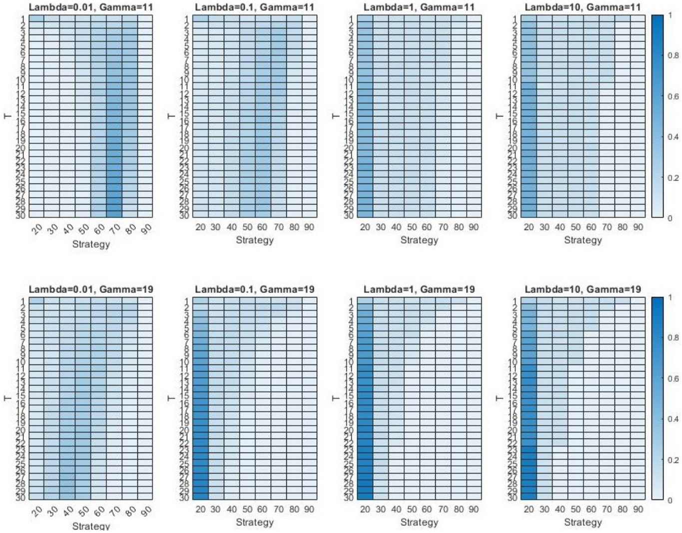
Two free parameters:

- stochastic sampling parameter λ
- time limit T

Decision rule:

Choose i if $A_i(T) = \max_{\imath'} A_{\imath'}(T)$ (ties broken randomly)

Traveler's Dilemma: Dual Accumulator Predictions



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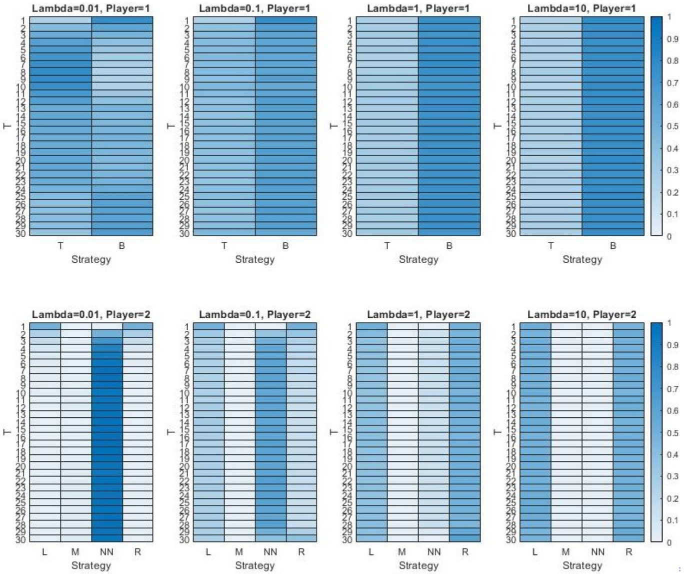
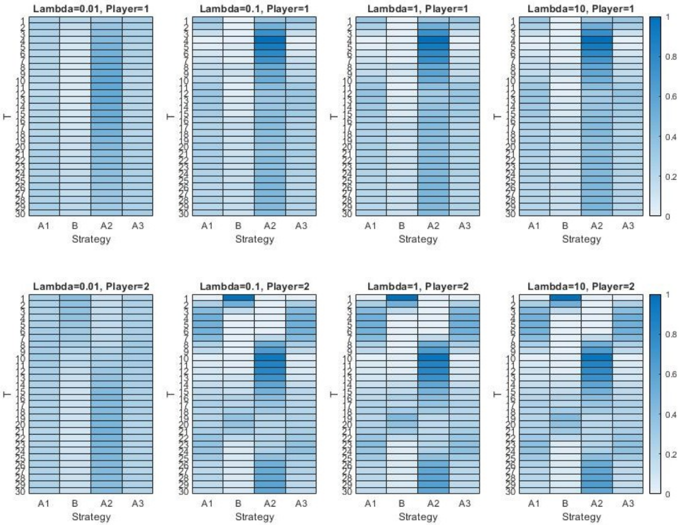


Hide and Seek: Dual Accumulator Predictions



Stag Hunt

	Hunter 2: Stag	Hunter 2: Hare
Hunter 1: Stag	100, 100	0, γ
Hunter 1: Hare	γ, 0	γ, γ

Nash Equilibrium Prediction

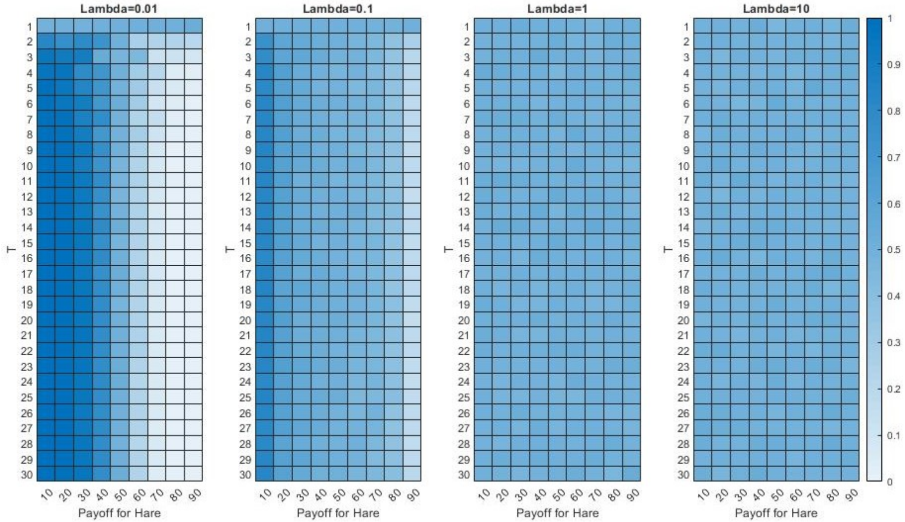
Either both hunt stag
or both hunt hare

Real Behavioral Pattern

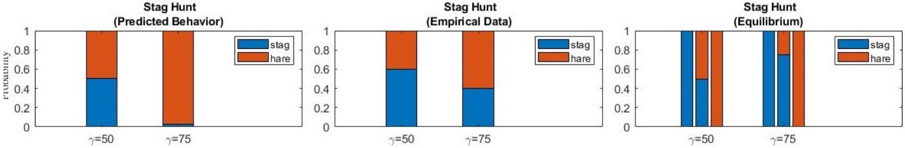
More hare as γ increases
Hare becomes modal for some γ
in $50 < \gamma < 100$

(Schmidt et al., 2003)

Stag Hunt: Dual Accumulator Predictions



Stag Hunt: Predictions vs Data



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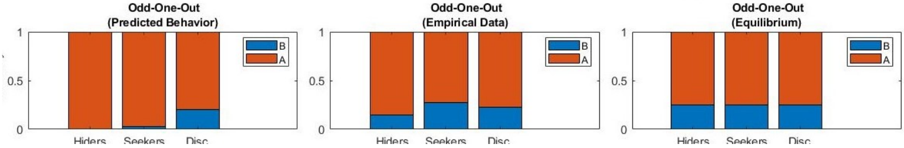
- Empirical data reported by Schmidt et al. (2003)

Odd-One-Out Games: Predictions vs Data



- Predicted behavior with $\lambda = .01$ and $T = 10$ and $\sigma_1 = \sigma_2 = [100, 0, 0, 0]$
- Empirical data reported by Hargreaves Heap et al. (2014)