THE BELL CURVE IS COUNTER-INTUITIVE

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Motivation and Background

Much cognitive psychology literature (Winkler, 1968; Hogarth, 1975; Edwards, 1968) focuses on how people update beliefs over time and examines belief updating in different domains (Moorthy, Ratchford, & Talukdar, 1997). Fewer investigations have looked into how people form mental representations prior to any belief updating, which is when their uncertainty about a domain is at its maximum.

In this research, we investigate:

Q1: How does the mental distribution look like when people consider a variable about which they possess little knowledge?

- The bell curve: Winkler (1967) postulate that people tend to generate a normal distribution for any variable they encounter
- The flat line: Fox and Cleman (2005) suggest that people have "ignorant prior" and generate uniform distribution when they have limited knowledge

Q2: How does this distribution evolve with knowledge?

These questions have theoretical significance to Bayesian models of information updating, as the original mental distribution can determine subsequent prediction functions and model performance (Griffiths & Tenenbaum, 2006).

Hypotheses

H1: People intuit a uniform distribution when they possess little knowledge about the variable

H2: Once people have a general idea (not actual specific knowledge) about that variable, they generate a distribution that is less flat and more bell-shaped, even though the actual distribution is either a bell curve or a flat line

Experimental Designs

Exp 1: Various Life Scenarios (M Turk, N = 387), Test H1

Task: read 6 scenarios out of 12 scenarios and allocate a certain number of items into several bins

No	Scenario	Attribute	Total Number of Items	Number of Bins
1	Movie (Rotten Tomato)	Rating	15940	10
2	Beatles	Song Length	100	12
3	Shrimp	Size	50	5
4	Star	Brightness	150	11
5	Apartment	Size	242	11
6	Commute	Time	64	8
7	Flight	Delay	90	8
8	Beer	ABV	30	6
9	Glasses	Prescription	120	12
10	French Croissants	Butteriness	50	5
11	Car	MPG	50	15
12	Credit Card Application	APR	50	9

In Exp 4 (not reported here), we also rule out the explanation that participants were mindlessly drawing flat distributions.

Exp 2: Song Length (M Turk, N = 481), Test H2, Pre-registered

IV: familiarity of song length (3 between-subjects conditions)
Haidt's Symphony (unfamiliar)
Beatles (familiar)
Hit Song (familiar)

Feature: controlling for actual distributions (as normal distributions)

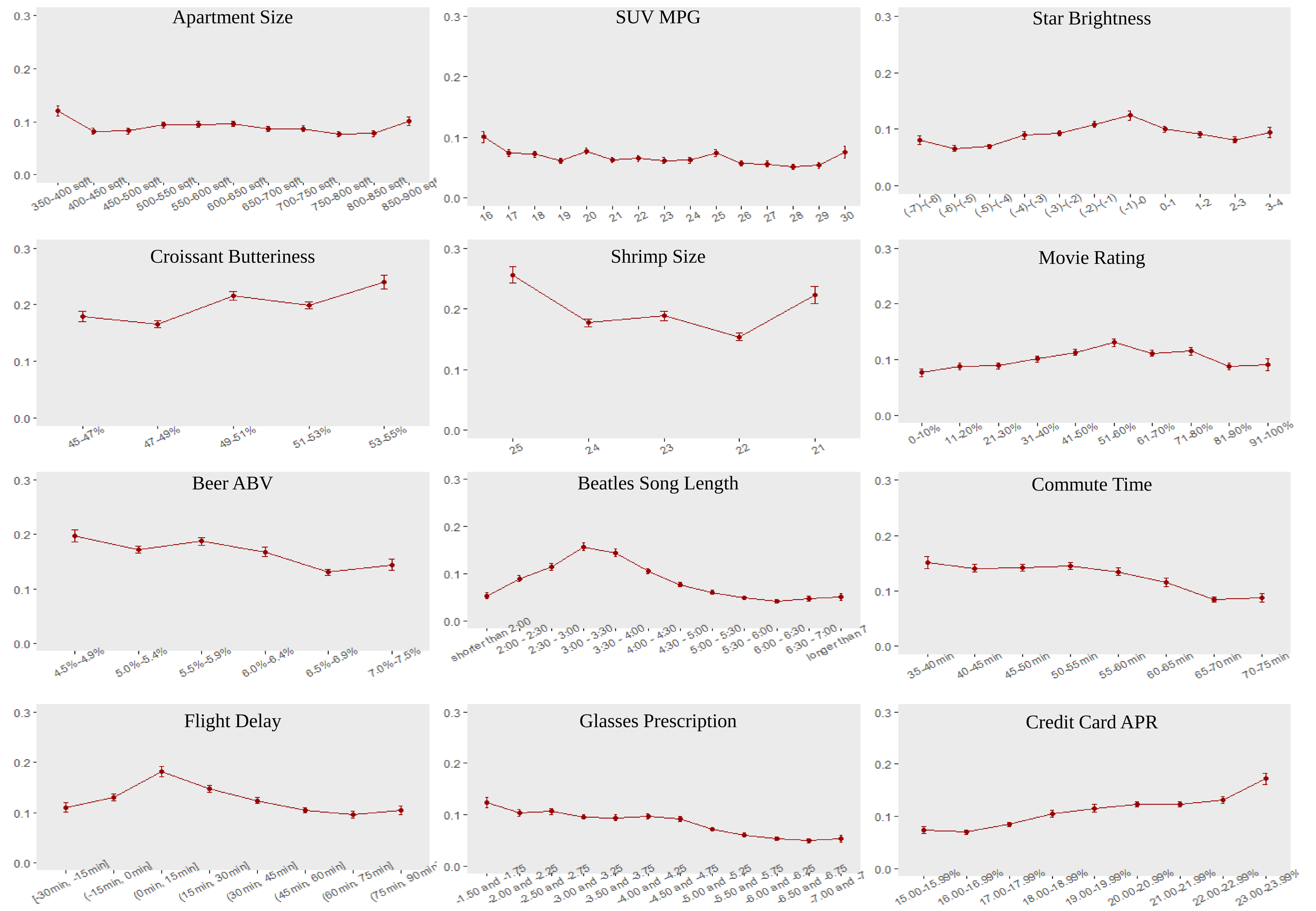
Exp 3: Credit Score (M Turk, N = 357), Test H2, Pre-registered

IV: actual distribution (3 between-subjects conditions)
young age (monotonously sloping down)
middle age (U shaped)
old age (monotonously sloping up)

Feature: incentivizing response accuracy

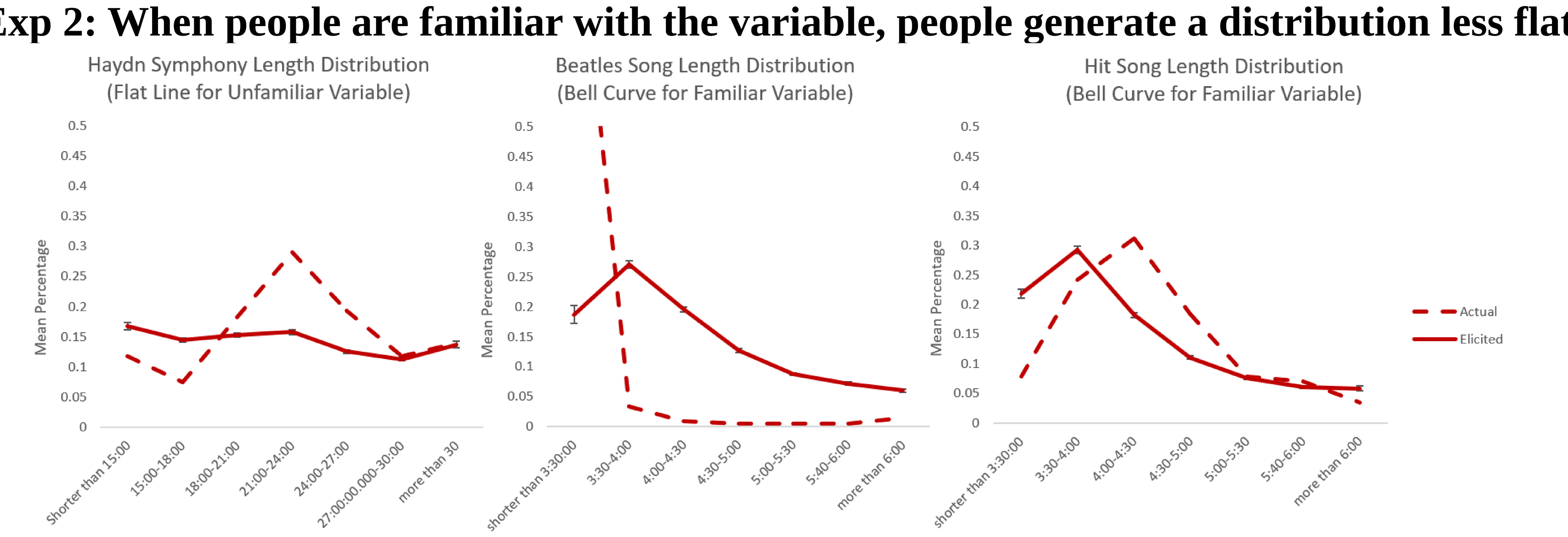
Our Findings

Exp 1: People do not intuit the bell curve. Instead, their intuitive distributions are rather flat.

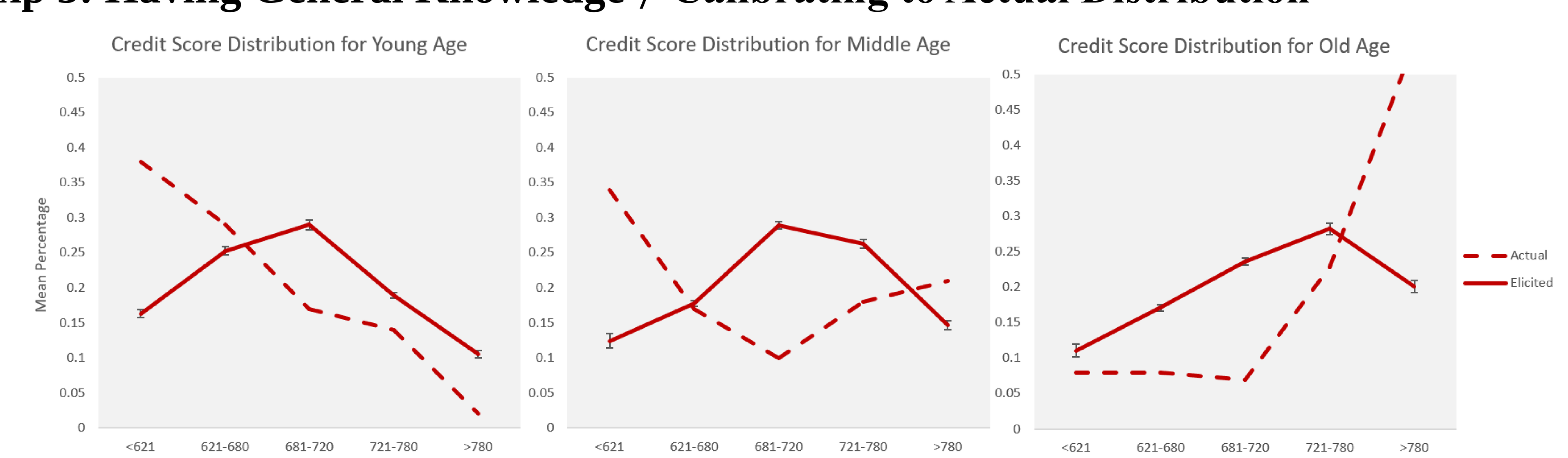


* Y axis is the mean percentage of all items allocated to each bin. X-axis shows the bin values. Error bars show the standard error.
* The depicted distribution is the aggregate distribution of all participants answering the same scenario, calculated by taking the mean percentage allocated to each bin. Data analysis are conducted on both the aggregate and individual levels, and the latter is reported here.

Exp 2: When people are familiar with the variable, people generate a distribution less flat



Exp 3: Having General Knowledge ≠ Calibrating to Actual Distribution



Data Analyses

Main Dependent Variable: Kurtosis of each individual's distribution

Kurtosis is the 4th central moment of a distribution. It measures the "tailedness" of a distribution. The kurtosis of any univariate normal distribution is 3, and distributions with kurtosis smaller than 3 (lower peak; thinner tails) are named *platykurtic* while kurtosis larger than 3 (higher peak; fatter tails) are *leptokurtic*.

Why kurtosis? Higher moments of the distribution better describes the shape of the distribution. In showing difference in kurtosis, we argue that participants' intuitive distribution are not Gaussian shaped with larger variance, but a different shape that should be simulated or approximated in a different way.

Exp 1: We calculated the kurtosis for each participant's elicited distribution for each scenario, and used t-test to compare the mean kurtosis for each scenario against uniform (kur = 1.8) and normal (kur = 3). All scenarios reject uniform distribution. Most scenarios show smaller than 3 kurtosis (platykurtic).

	Kurtosis Statistics		Test of Kurtosis Against 3 (Normal)	
	Percentage of Kurtosis < 3	Median Kurtosis	T-statistic	P Value
Beatles Song Length	53.65%	2.764	3.22	NS (more peaked than normal)
Flights Delay	71.35%	2.353	-1.75	Reject, p<0.05(=0.041)
SUV MPG	72.49%	2.369	-2.94	Reject, p<0.01
Glasses Prescription	73.37%	2.344	-1.10	p=0.14
Movie Rating	73.96%	2.331	-1.19	p=0.12
Credit Card APR	76.65%	2.355	-4.29	Reject, p<0.001
Commute Time	78.68%	2.301	-2.03	Reject, p<0.05(=0.022)
Apt Size	78.95%	2.222	-5.77	Reject, p<0.001
Beer ABV	83.25%	2.1	-3.37	Reject, p<0.001
Star Brightness	85.86%	2.2	-2.31	Reject, p<0.05(=0.011)
Shrimp Size	89.01%	1.938	-2.63	Reject, p<0.01
Croissant Butteriness	90.77%	1.972	-6.61	Reject, p<0.001

* Kurtosis for normal distribution is 3 and for uniform distribution is 1.8.

Exp 2: Mean kurtosis is 4.08 for the Hit song condition, 4.55 for the Beatles song condition and 2.32 for the Symphony condition. The difference between Hit & Symphony and Beatles & Symphony is significant (t = 8.27 and 3.92, $p < 0.001$), while the difference between Hit and Beatles is not significant (t = 0.81, $p = .42$).

Exp 3: The average kurtosis for each condition is not different from each other (kur = 2.47, 2.52, 2.64 for Young, Middle and Old respectively, all *n.s.*). All elicited distribution were different from actual distribution: Young: t = 15.40, $p < .001$; Mid: t = 2.63; $p < .005$; Old: t = -7.22 (elicited is less peaked), $p < .001$.

Directions

- Show that such belief is not restricted to the distribution elicitation method.
  Ongoing Lab Study: participants allocate items one by one.
- Investigate the behavioral consequences that emerge from the belief.
- Estimate properly shaped prior distribution and simulate posterior belief distribution from the estimation. Test if the simulated posterior is descriptive of people's actual judged posterior.

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