Lay Understanding of Outliers

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SJDM 2017

The Story of Mr. and Mrs. Hadlum

Barnett, 1978

The Story of Mr. and Mrs. Hadlum

(specifically, the story of their divorce)

Barnett, 1978

Barnett, 1978

January

S	M	T	W	T	F	S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
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February

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May

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Barnett, 1978

baby born

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Barnett, 1978

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349 days earlier!

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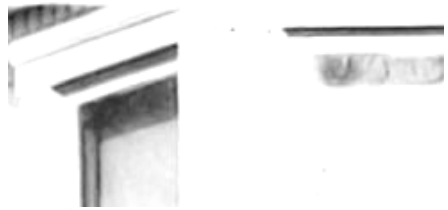
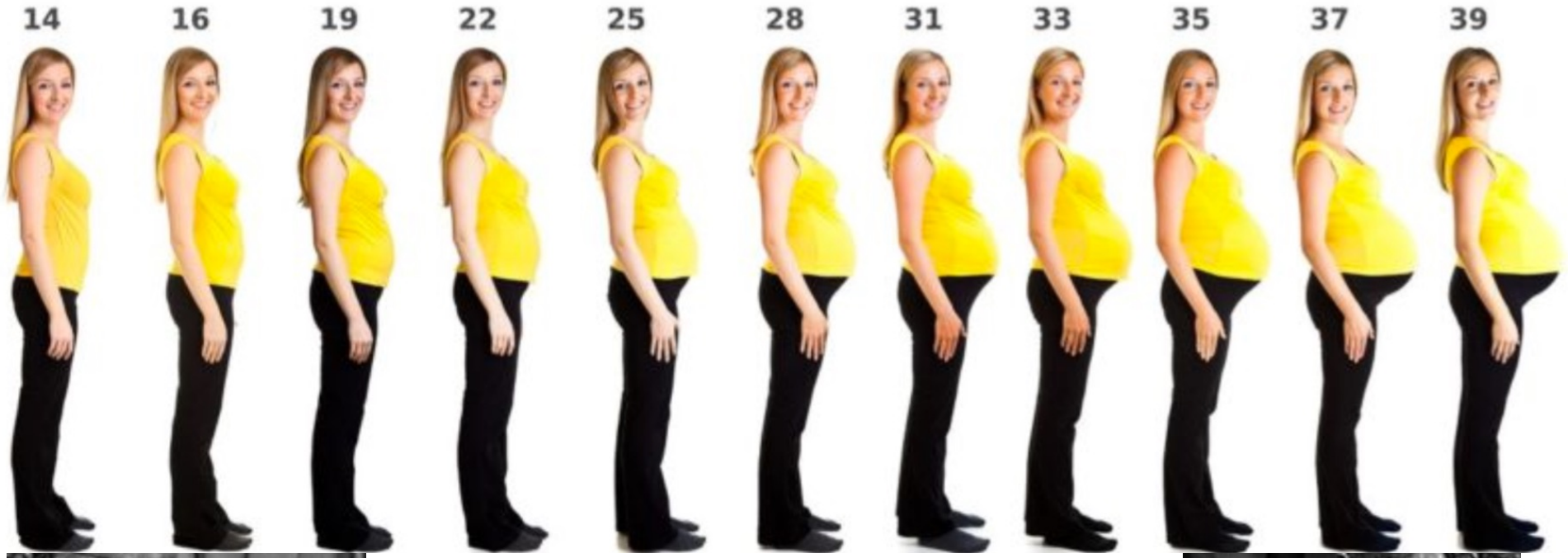
Barnett, 1978

Barnett, 1978





… and then about **10 more weeks** before Mrs. Hadlum gives birth

Barnett, 1978

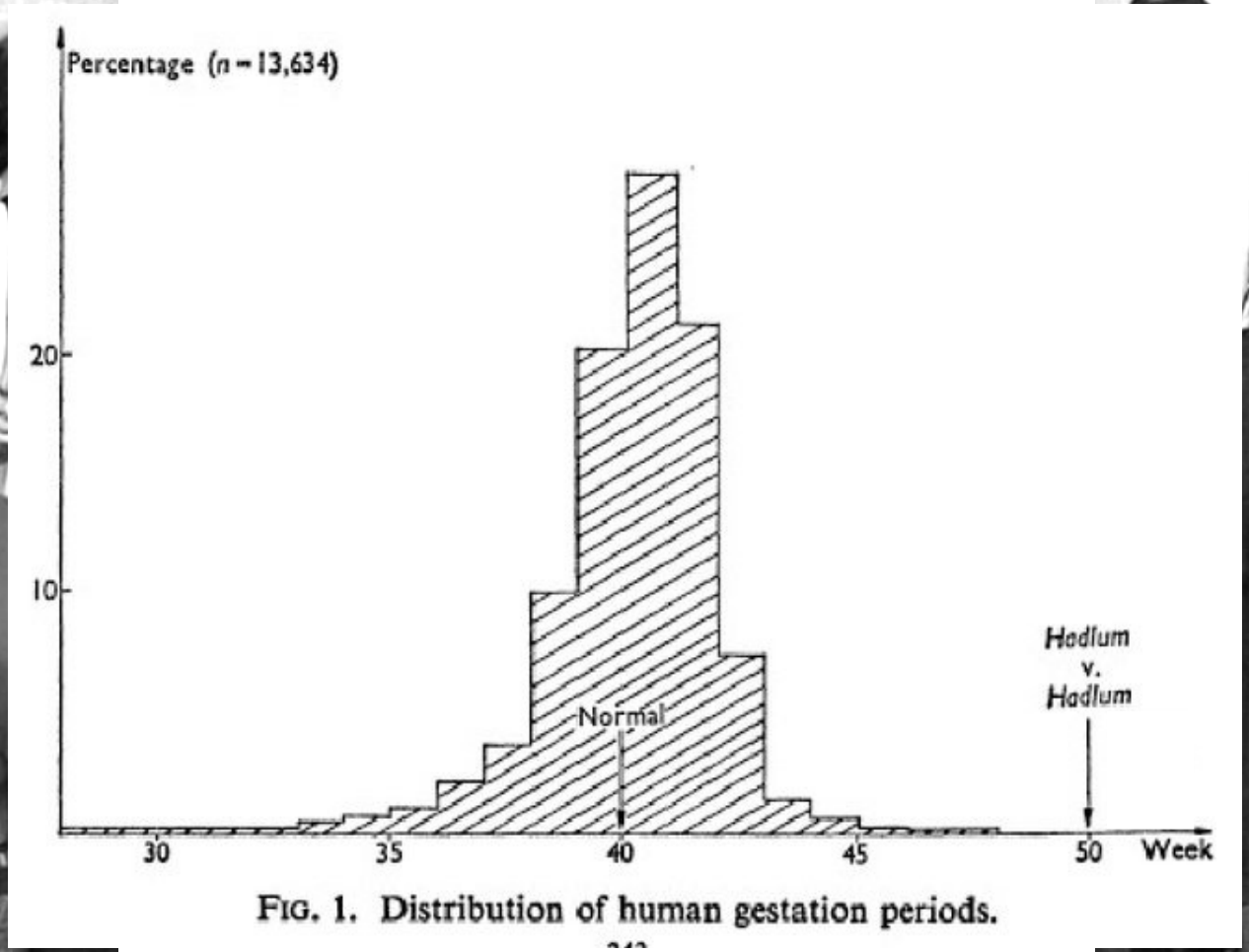


FIG. 1. Distribution of human gestation periods.

Barnett, 1978

14Ginola
Halifax, United Kingdom

Reviewer

3 reviews

3 hotel reviews

14 helpful votes

"Not as bad as reviews state."

Reviewed 4 weeks ago

For what you pay for this hotel it's fine. not half as bad as reviews state. It's typically 70's furniture, flowery wallpaper, blown vinyl, toilets are of 70's colours. But then bed linen was clean & comfy. Down stairs was cosy with a nice little bar. Cheap drinks & friendly staff who know they are not running the Hilton &...

More

Was this review helpful? Yes 19

14Ginola
Halifax, United Kingdom

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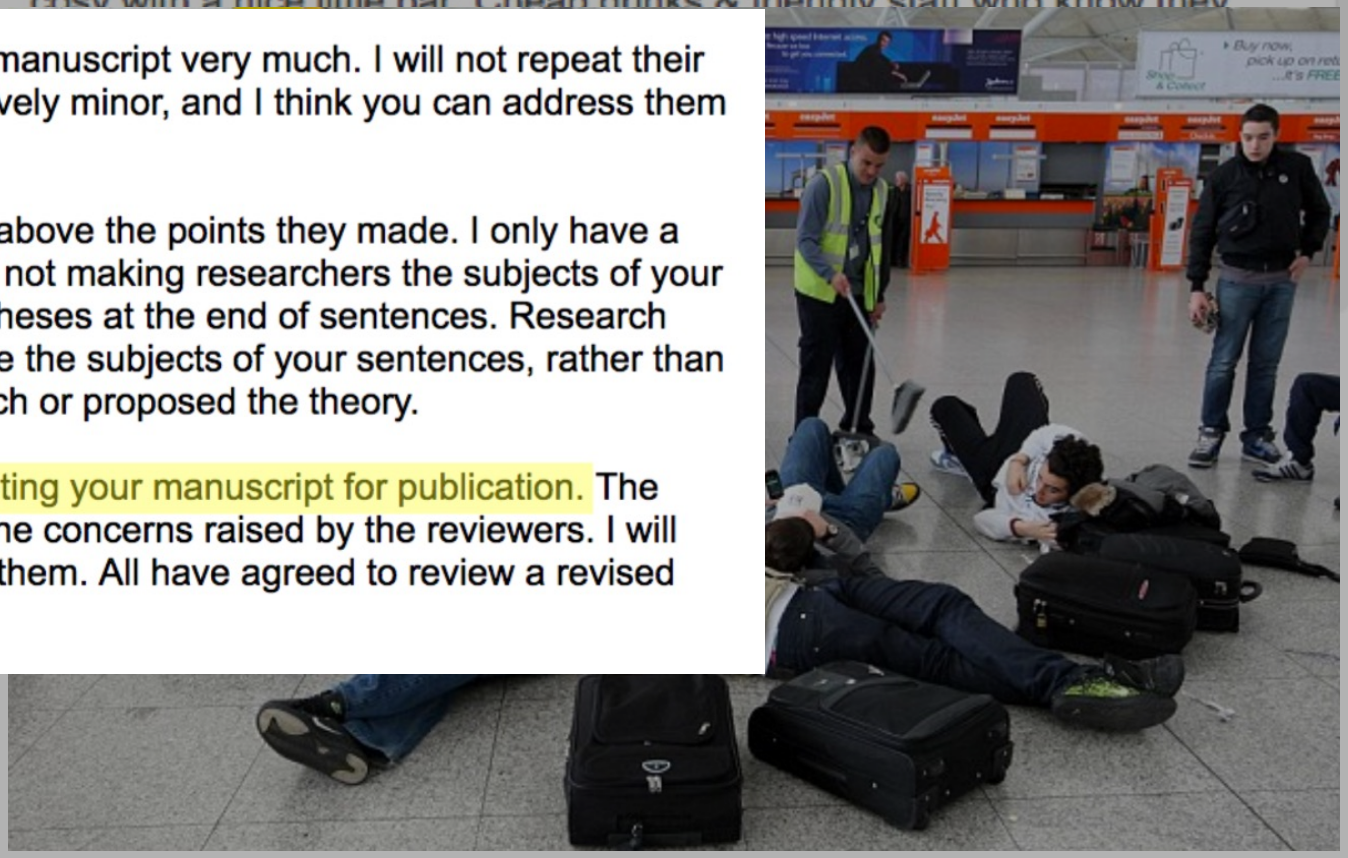
Like the reviewers, I like your manuscript very much. I will not repeat their concerns here. They are relatively minor, and I think you can address them in a revision.

I actually have nothing to add above the points they made. I only have a minor comment. I recommend not making researchers the subjects of your sentences. Put them in parentheses at the end of sentences. Research findings and theories should be the subjects of your sentences, rather than the people who did the research or proposed the theory.

Thus, I am conditionally accepting your manuscript for publication. The condition is that you address the concerns raised by the reviewers. I will send your manuscript back to them. All have agreed to review a revised revision.



“an observation which deviates so much from other observations as to cause suspicions that it was generated by a different mechanism.”

(Hawkins, 1980)

“an observation which deviates so much from other observations *as to cause suspicions that it was generated by a different mechanism.”*

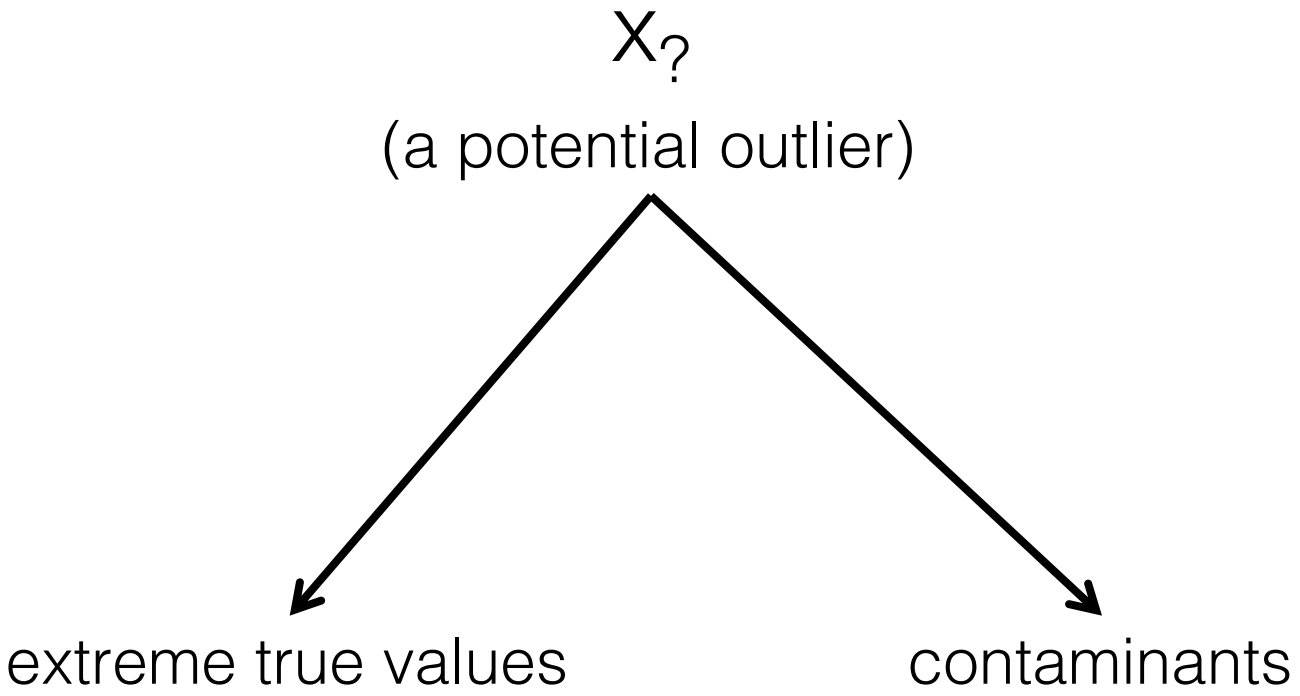
(Hawkins, 1980)

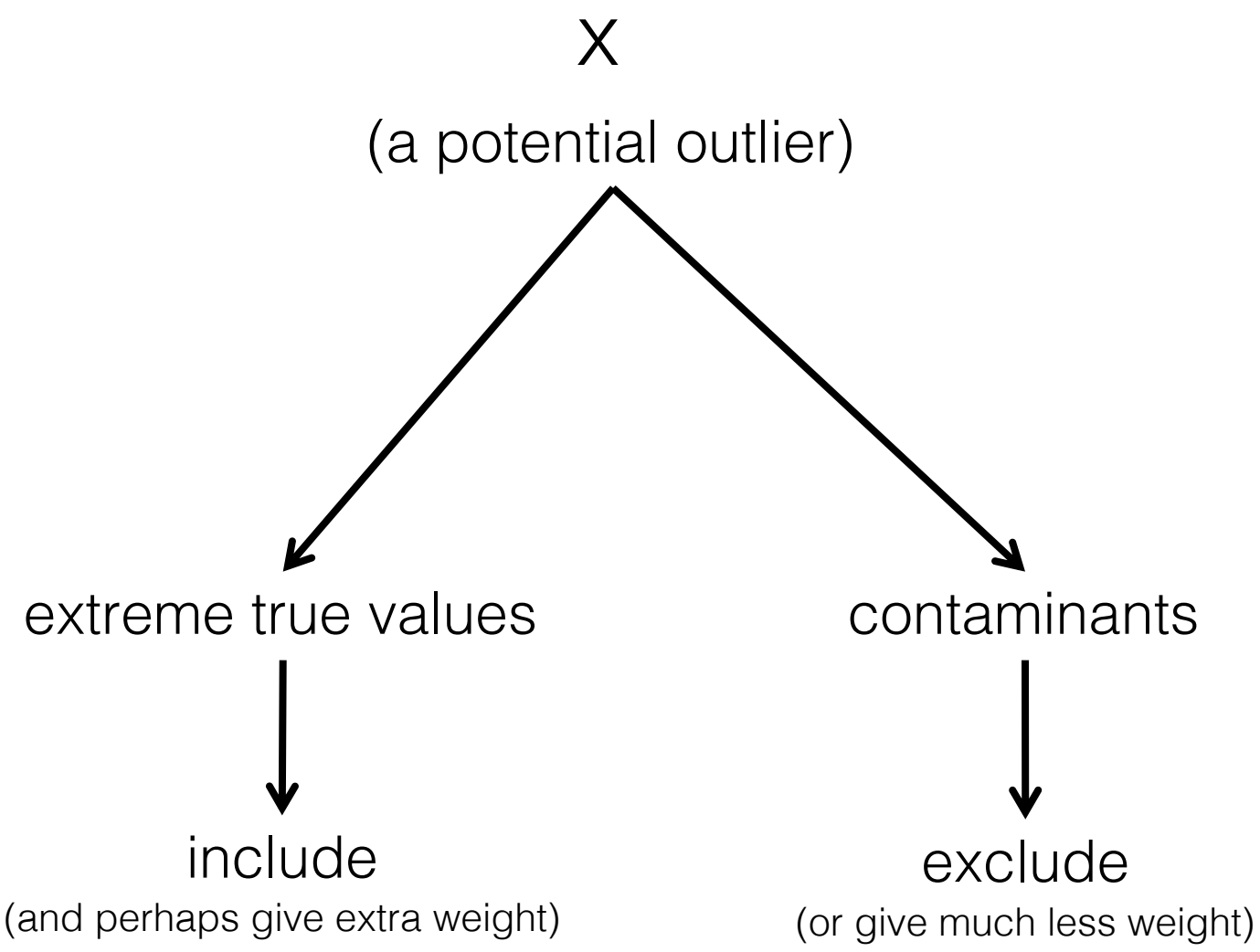
“an observation which **deviates** so much from other observations as to cause suspicions that it was generated by a different mechanism.”

(Hawkins, 1980)

$X_?$

(a potential outlier)


$x_?$
(a potential outlier)
extreme true values
contaminants


X
(a potential outlier)
extreme true values
contaminants
include
(and perhaps give extra weight)
exclude
(or give much less weight)

How do individuals incorporate outliers from a sample into their predictions of the population distribution?

the task

a small sample of some data with an *potential outlier*

$(x_1, x_2 \ldots x_{n-1}, x_n)$

evaluation & weighting

prediction of the population mean

How might participants approach this?

Directional predictions – overweighting

(Tversky & Kahneman, 1974; Lichtenstein, et al., 1978; Sunstein & Zeckhauser, 2011; Rothman, Klein, & Weinstein, 1996; Cruciani, Berardi, Cabib, & Conversi, 2011; Brown & Kulik, 1977; Christianson & Loftus, 1987; Madan, Ludvig, & Spetch, 2014; Ludvig, Madan, & Spetch, 2014).

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Use sample statistics (e.g. mean, median)

(Goldstein & Rothschild, 2014; Griffiths & Tenenbaum, 2006; Leider, Griffiths & Hsu, 2017)

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Use sample statistics but discount eventually

(Obrecht, Chapman & Suárez, 2010; Obrecht, Chapman & Gelman, 2007)

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Statistical benchmark - Tests of discordancy

How might participants approach this?

Directional predictions – overweighting

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Statistical benchmark - Tests of discordancy

$$T = \frac{excess}{outlier} = \frac{x_{(n)} - x_{(n-1)}}{x_{(n)}}$$

if sample is exponential distribution

(Likes, 1966)

$$T = \frac{excess}{range} = \frac{x_{(n)} - x_{(n-1)}}{x_{(n)} - x_{(1)}}$$

if sample is normal distribution

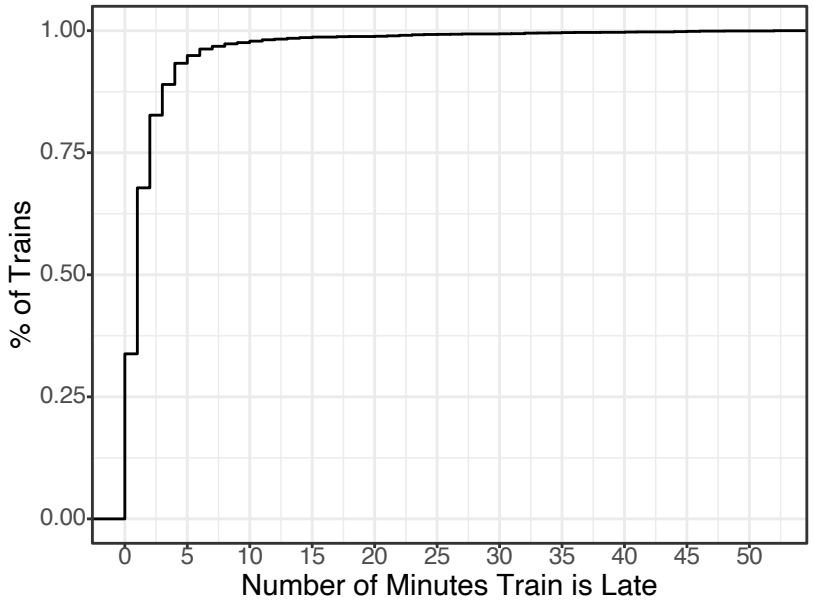
(Dean & Dixon, 1951)

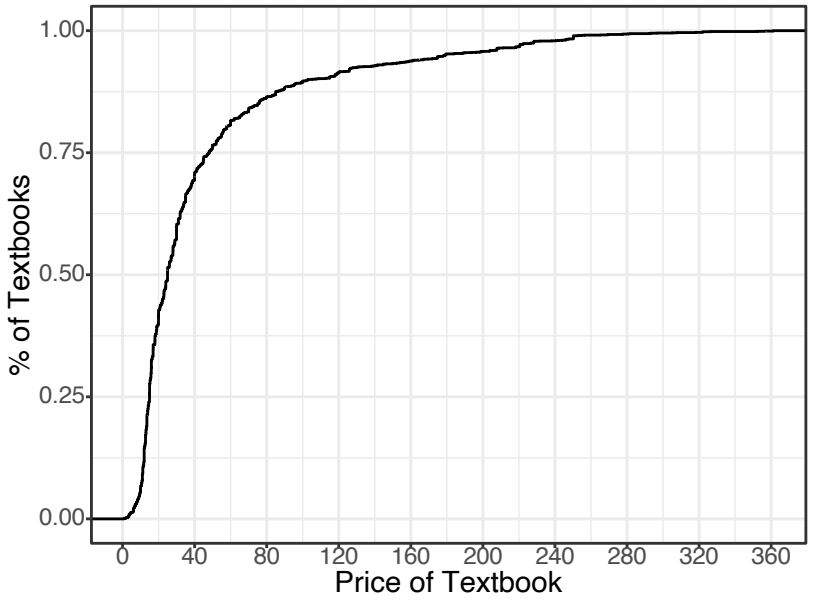
our contexts

our contexts

~3500 Train Arrival Times
from week proceeding survey

~2500 Textbook Prices
from quarter survey was offered





our procedure

IV: latest train
manipulated at 10 levels
from 6 minutes to 54 minutes

IV: most expensive book
manipulated at 9 levels
from $36.95 to $361.25

our procedure

IV: latest train
manipulated at 10 levels
from 6 minutes to 54 minutes

5 minutes
54 minutes
2 minutes
0 minutes
3 minutes

IV: most expensive book
manipulated at 9 levels
from $36.95 to $361.25

$361.25
$24.00
$20.00
$19.00
$29.00
$16.95

our procedure

IV: latest train
manipulated at 10 levels
from 6 minutes to 54 minutes

5 minutes
54 minutes
2 minutes
0 minutes
3 minutes

Based on this week, when do you think the train will arrive, on average, next week?

IV: most expensive book
manipulated at 9 levels
from $36.95 to $361.25

$361.25
$24.00
$20.00
$19.00
$29.00
$16.95

Based on this sample, how much would you say a single course book costs, on average?

our procedure

IV: latest train
manipulated at 10 levels
from 6 minutes to 54 minutes

5 minutes
54 minutes
2 minutes
0 minutes
3 minutes

Based on this week, when do you think the train will arrive, on average, next week?

199 participants
995 predictions

IV: most expensive book
manipulated at 9 levels
from $36.95 to $361.25

$361.25
$24.00
$20.00
$19.00
$29.00
$16.95

Based on this sample, how much would you say a single course book costs, on average?

214 participants
1070 predictions

analytic approach

Robust Regression

as a linear mixed-model with random participant intercepts

analytic approach

Robust Regression

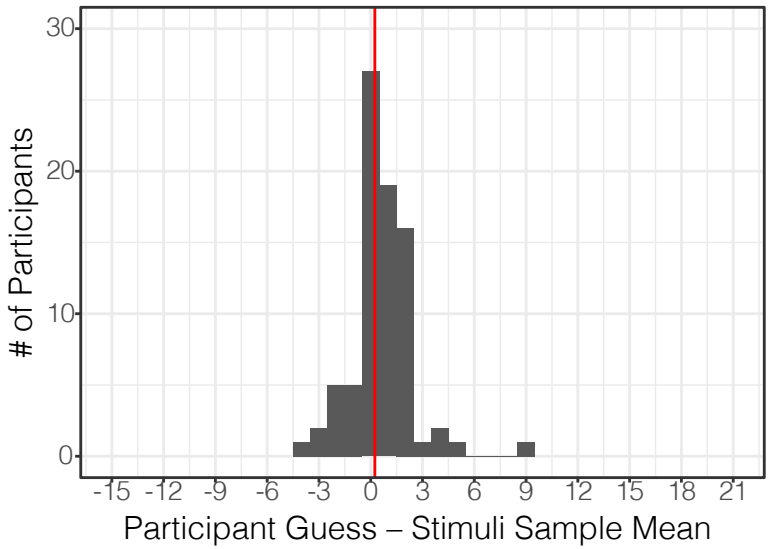
as a linear mixed-model with random participant intercepts

Quantile Regression

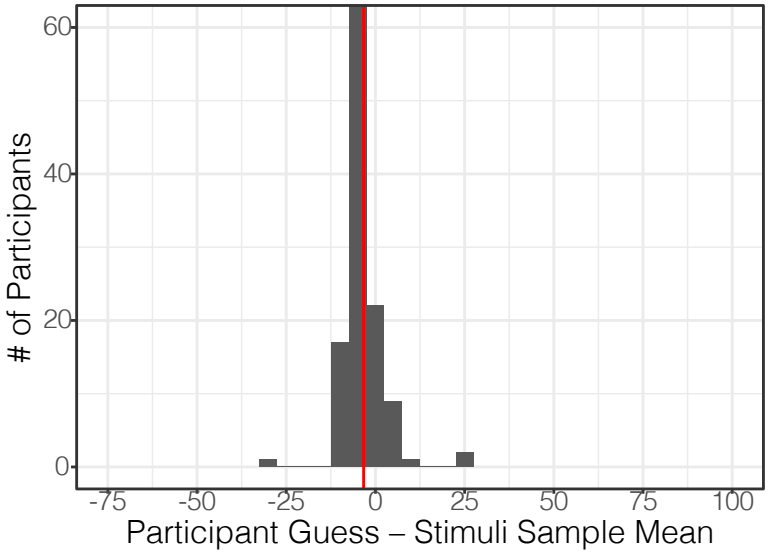
with bootstrapped SEs clustered by participant

predicting the 10^{th}, 50^{th} and 90^{th} percentiles

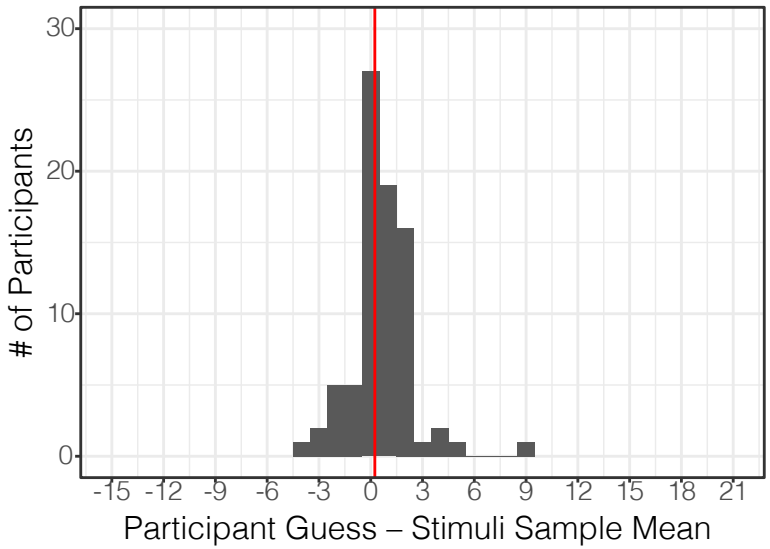
Latest Train is 12 minutes



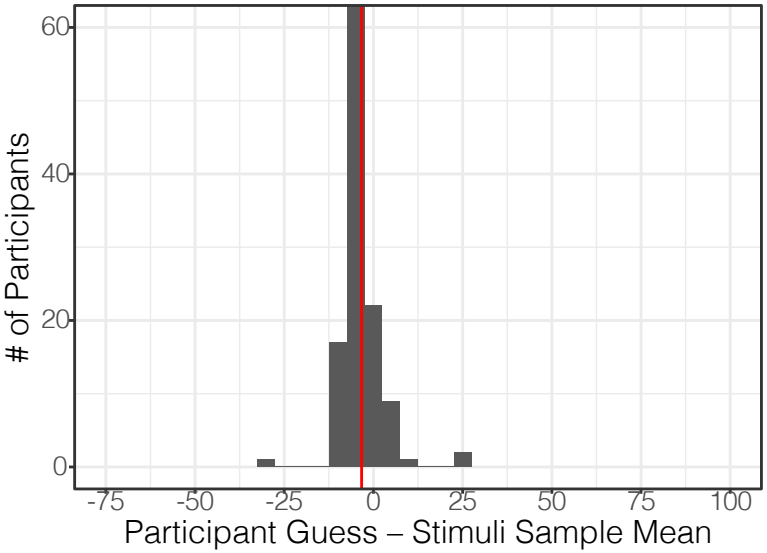
Most Expensive Book is $53.50



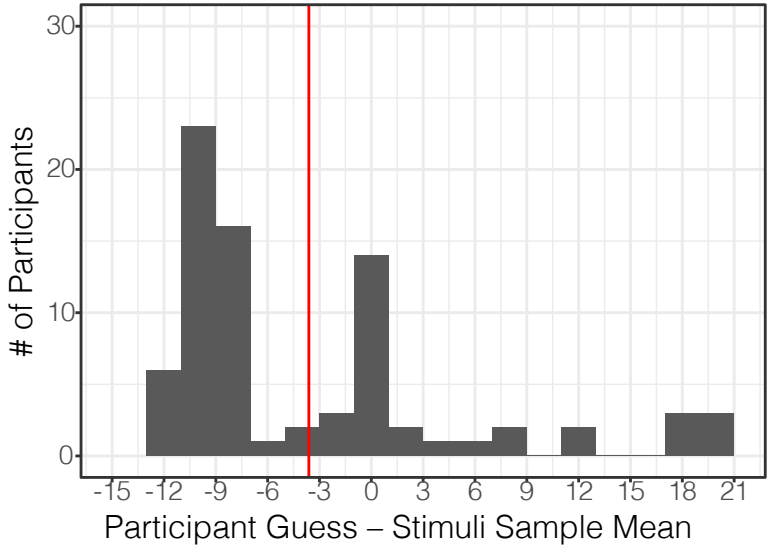
Latest Train is 12 minutes



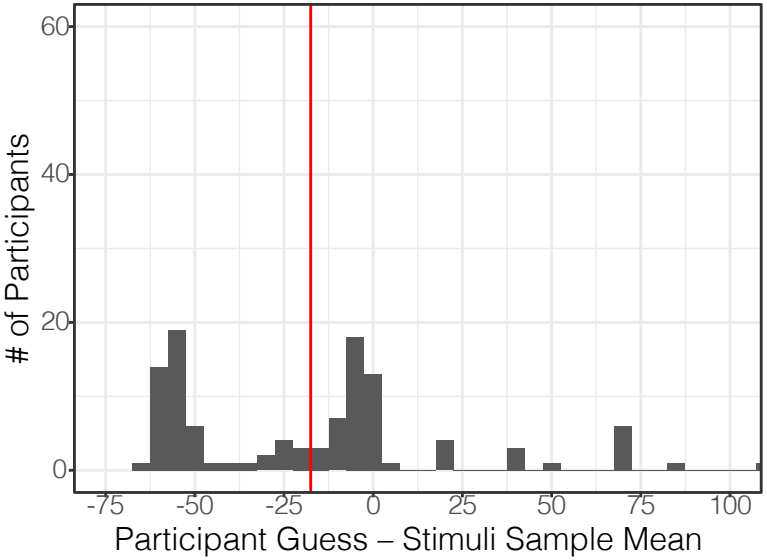
Most Expensive Book is $53.50



Latest Train is 54 minutes



Most Expensive Book is $361.25



results

Evaluate participants against 4 predictions:

1. Overweighting
2. Sample mean
3. Discordancy tests
4. Sample median

Is there evidence of giving outliers extra weight?

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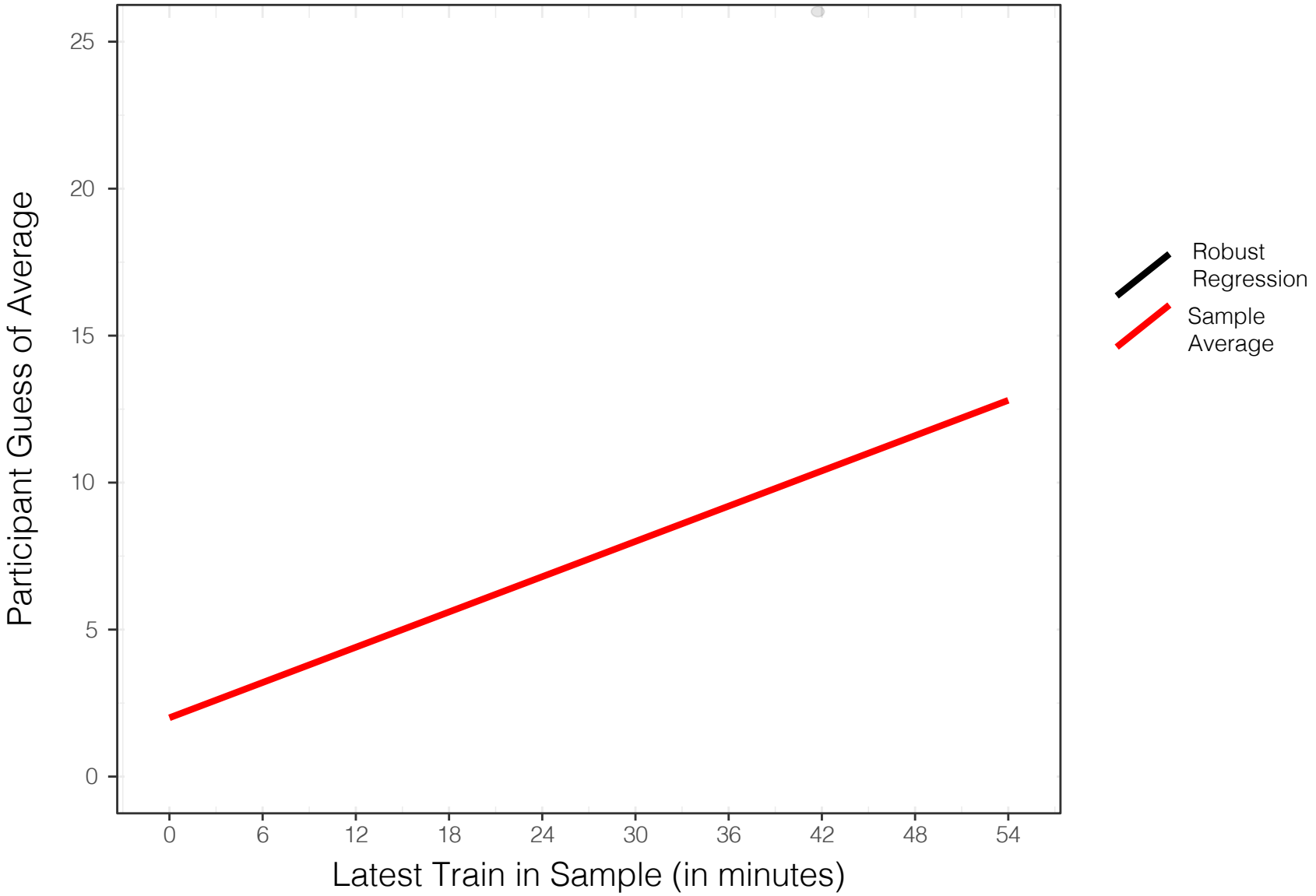

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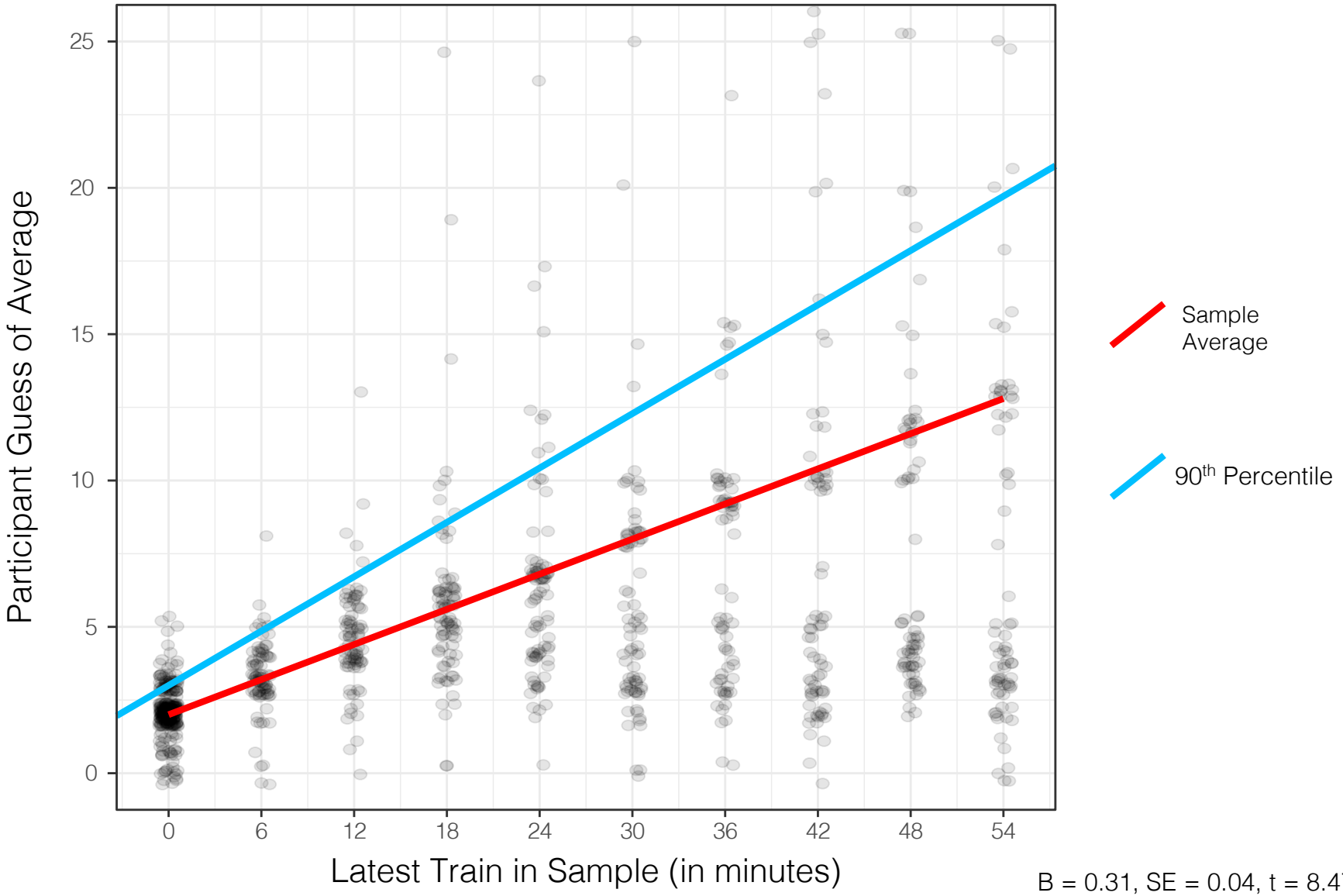
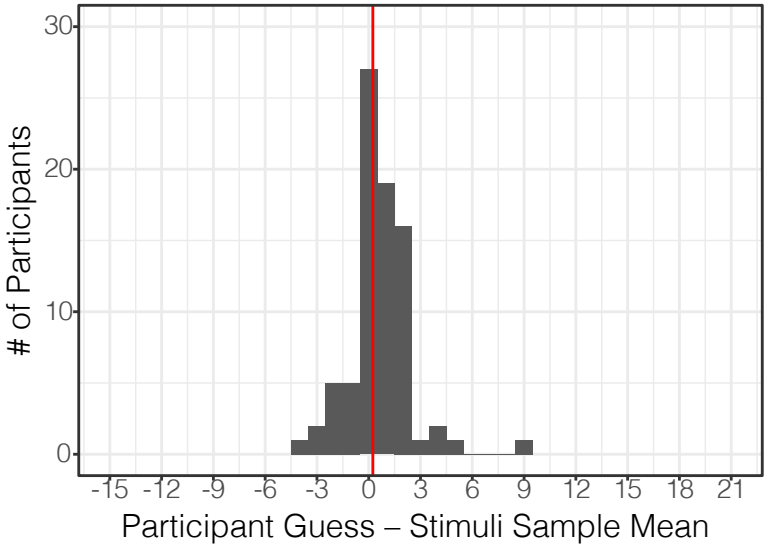


B = 0.31, SE = 0.04, t = 8.47

Latest Train is 12 minutes



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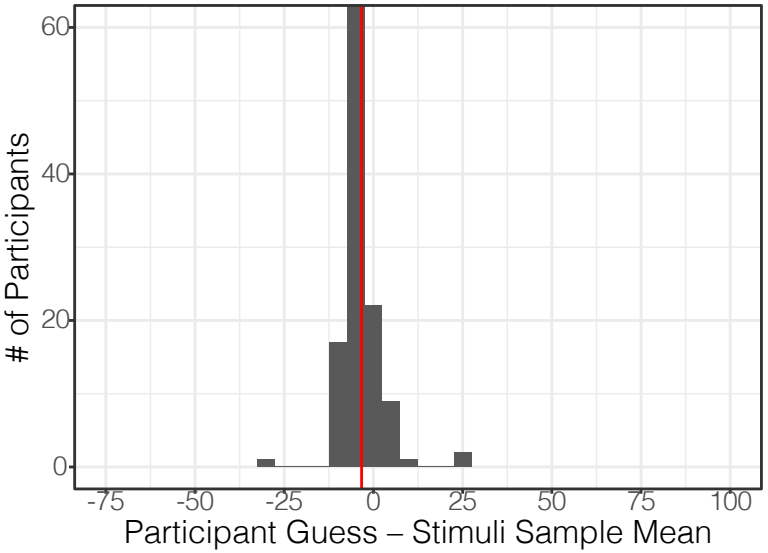

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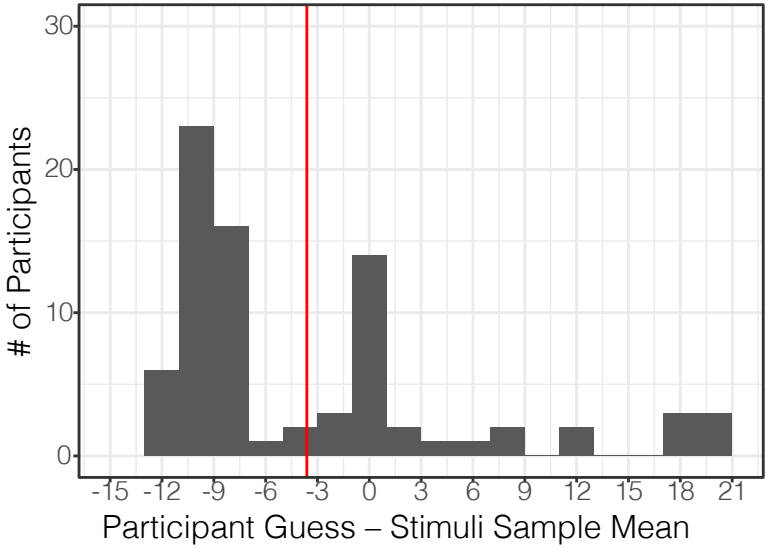

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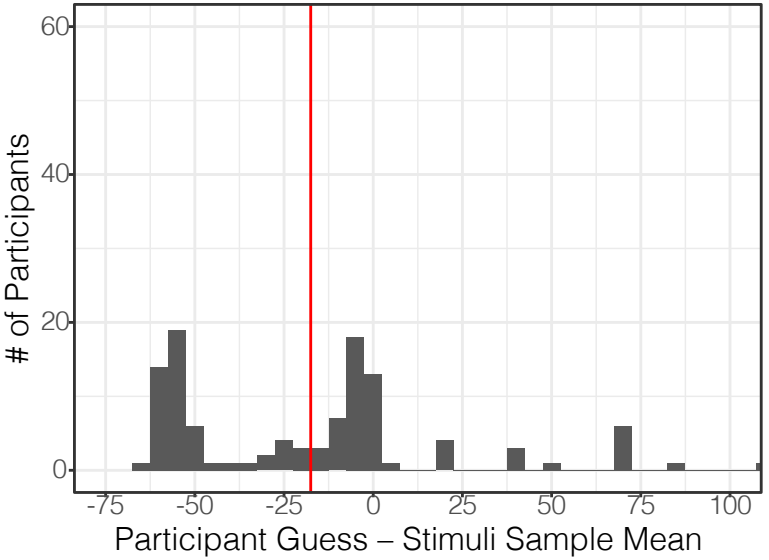
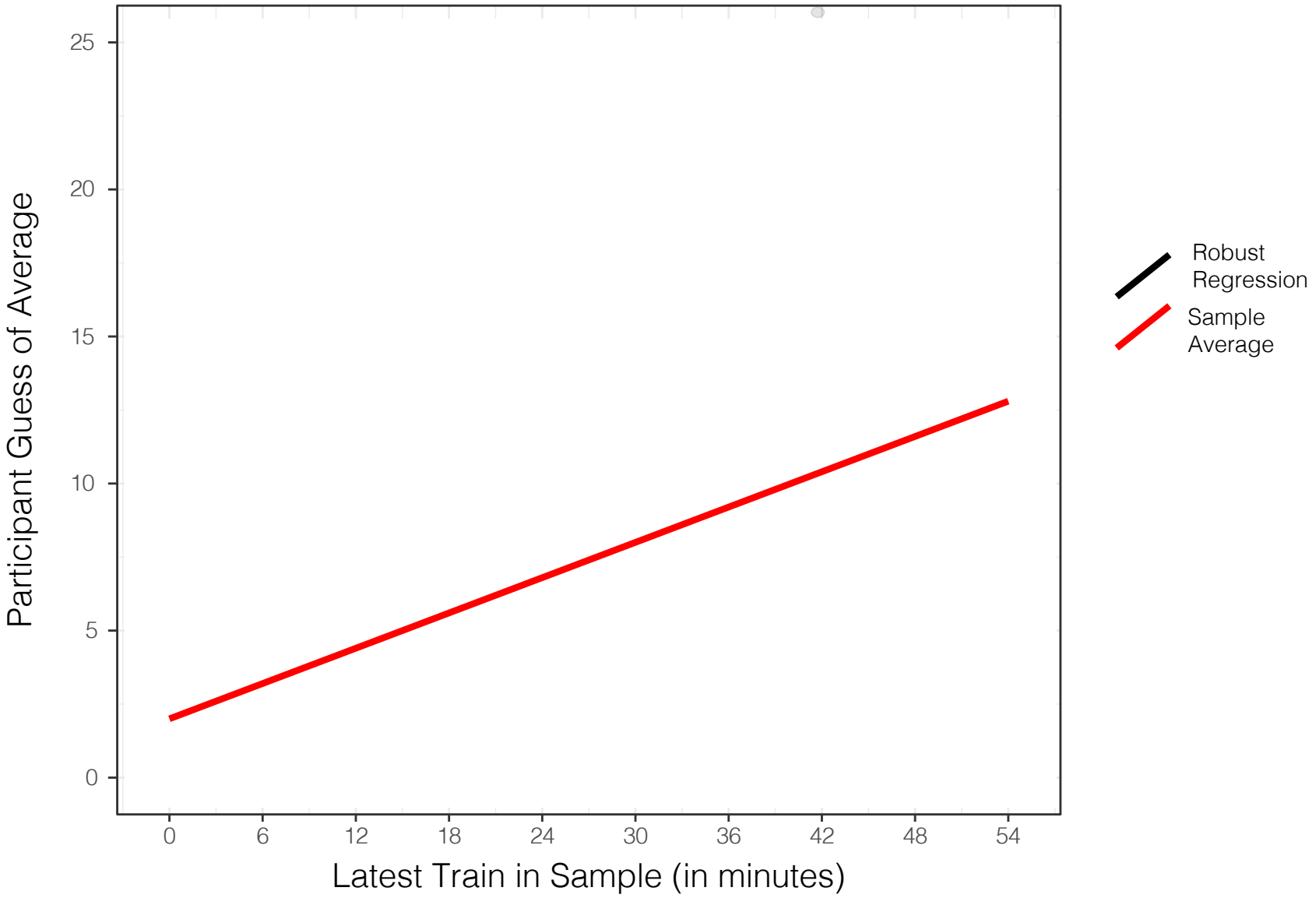


Do participants use the sample mean to predict population mean?



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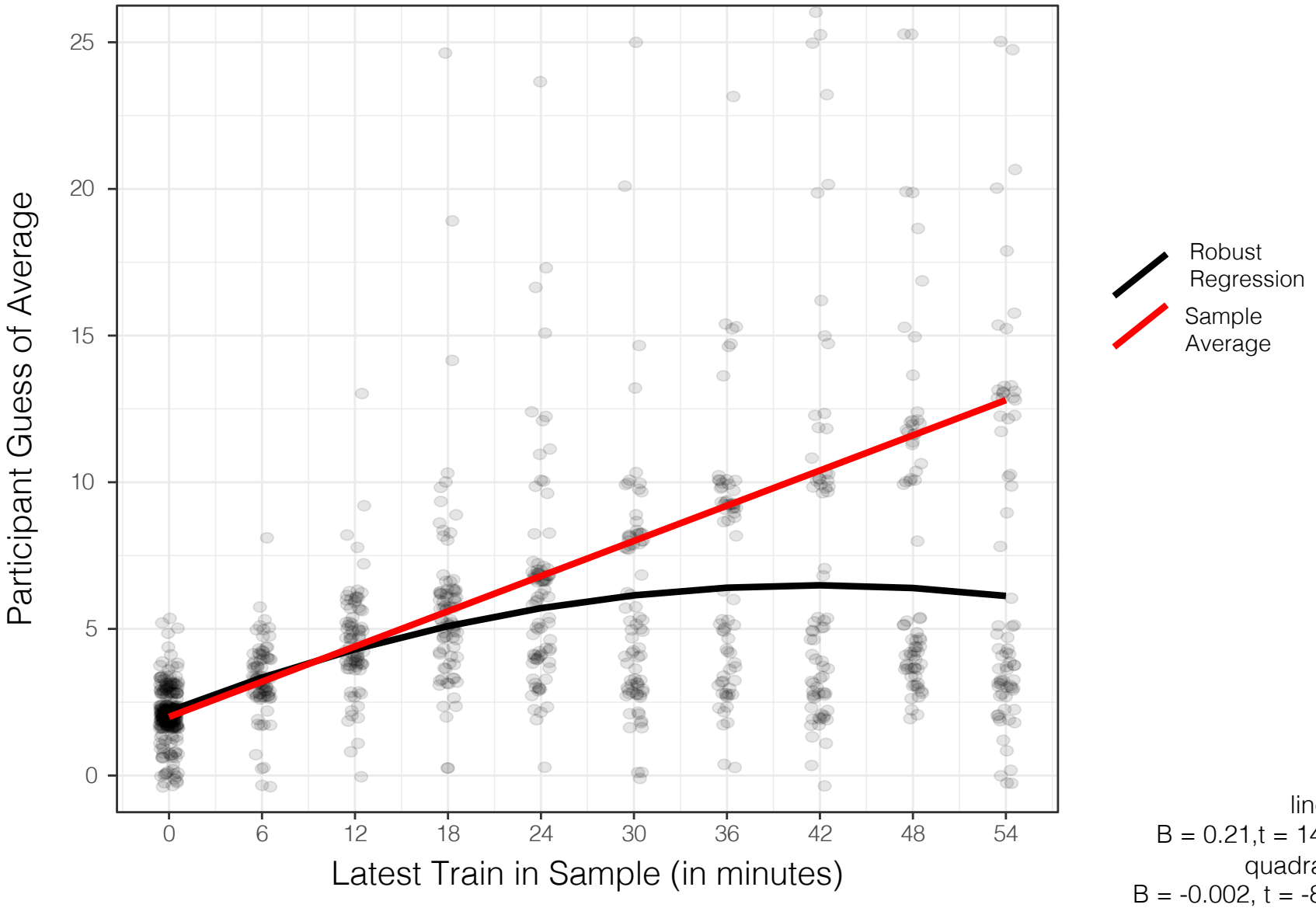
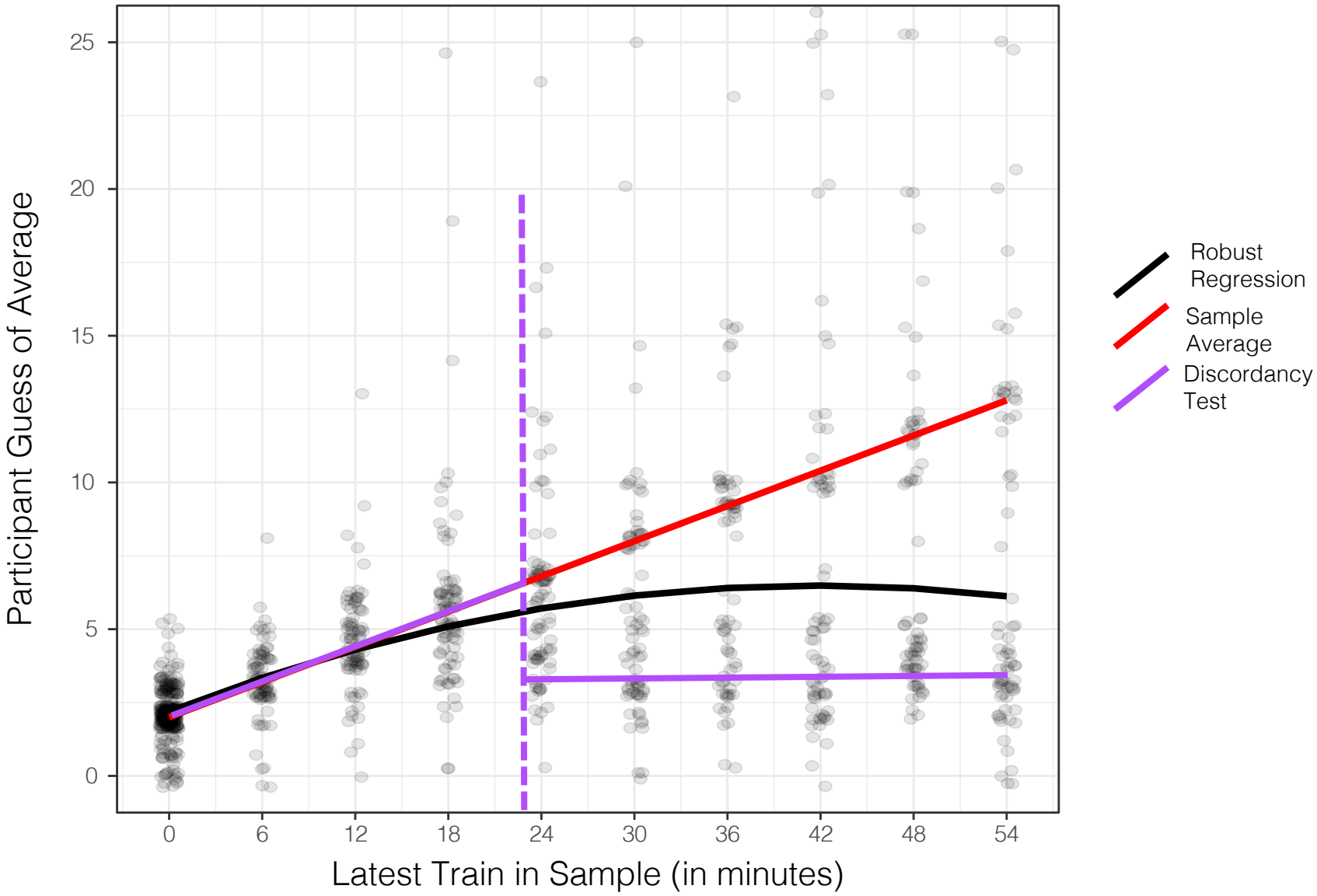


linear:
B = 0.21,t = 14.75
quadratic:
B = -0.002, t = -8.88

When do participants (on average) start to discount outliers?



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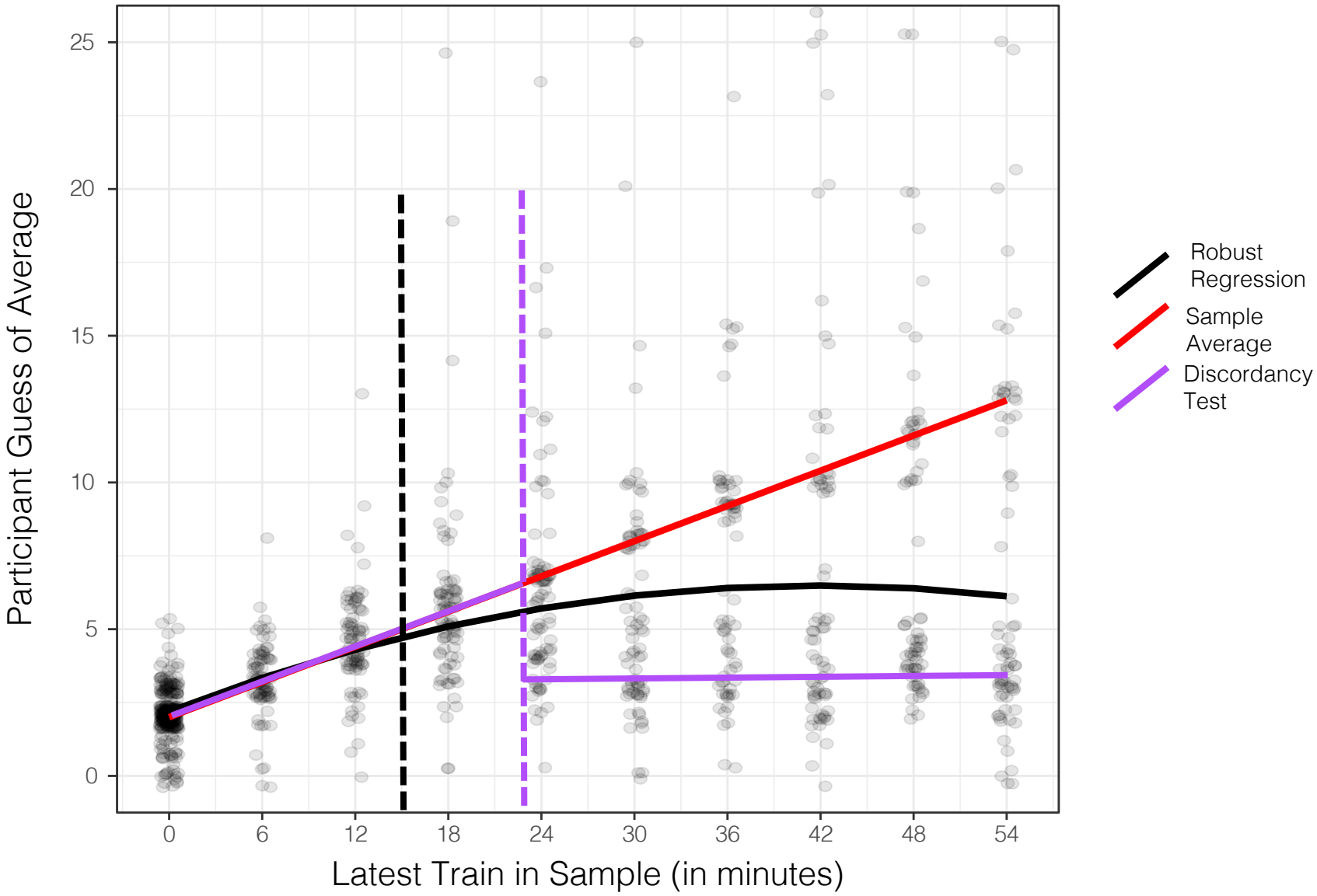

Do participants use the sample median to predict population mean?

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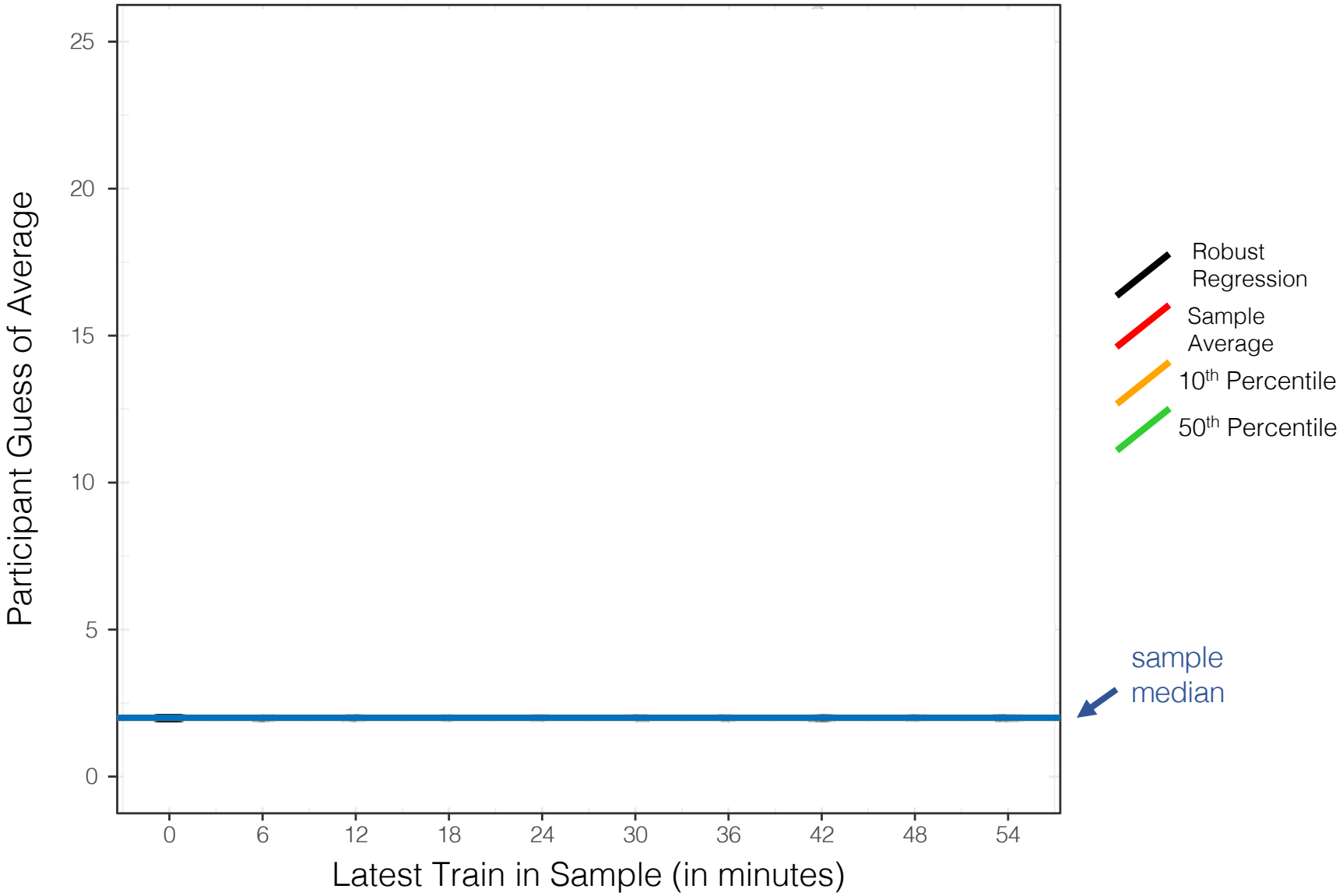

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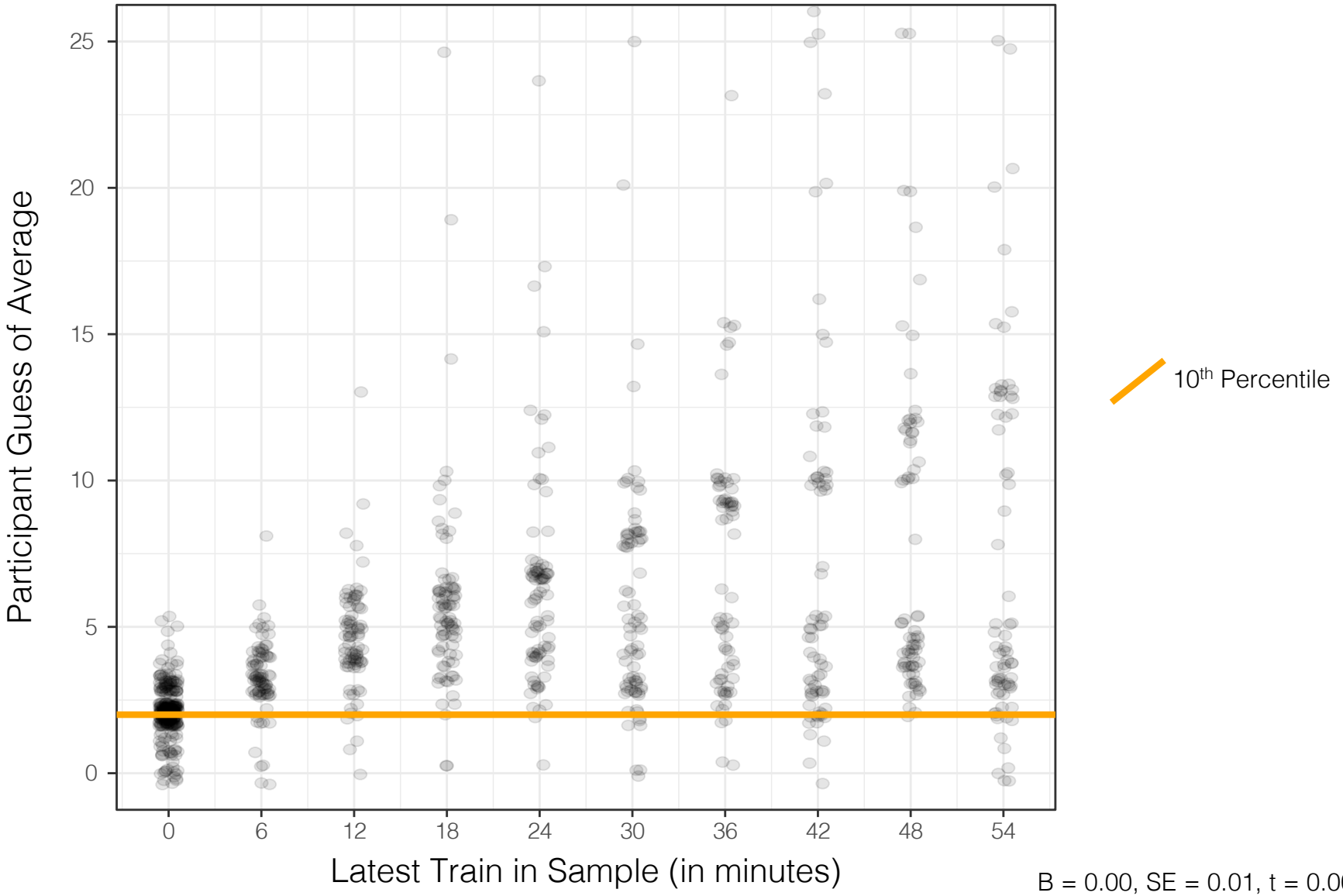

B = 0.00, SE = 0.01, t = 0.00

summary

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- about 10% of participants give extra weight to outliers in a sample
- about 10% of participants also always ignore outliers

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- other distributions?

summary

- about 10% of participants give extra weight to outliers in a sample
- about 10% of participants also always ignore outliers
- On average, participants start to discount early, but don't discount *enough*

open questions

- other distributions?
- individual differences in outlier appraisal?

Barnett, 1978

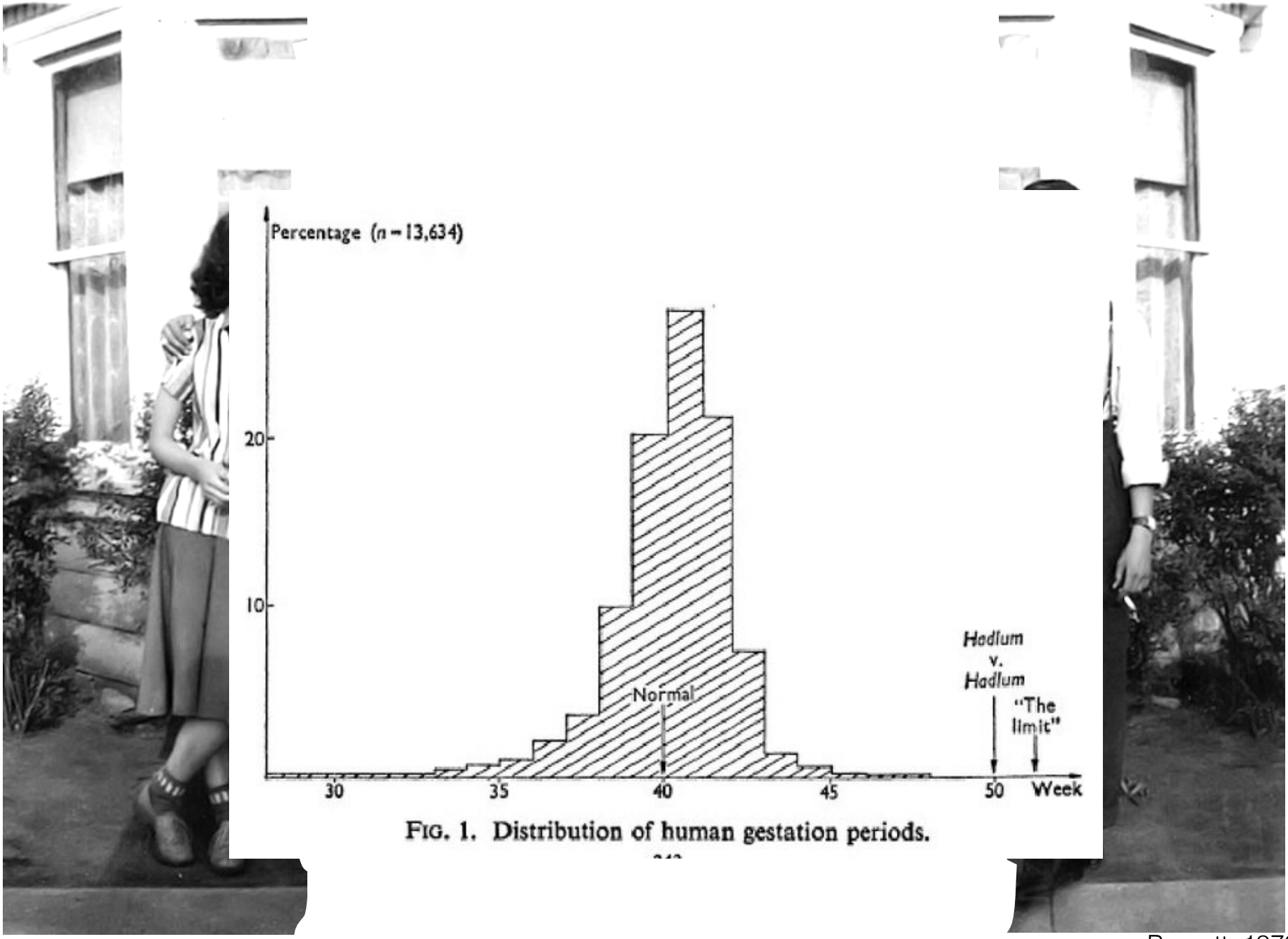

FIG. 1. Distribution of human gestation periods.

Barnett, 1978

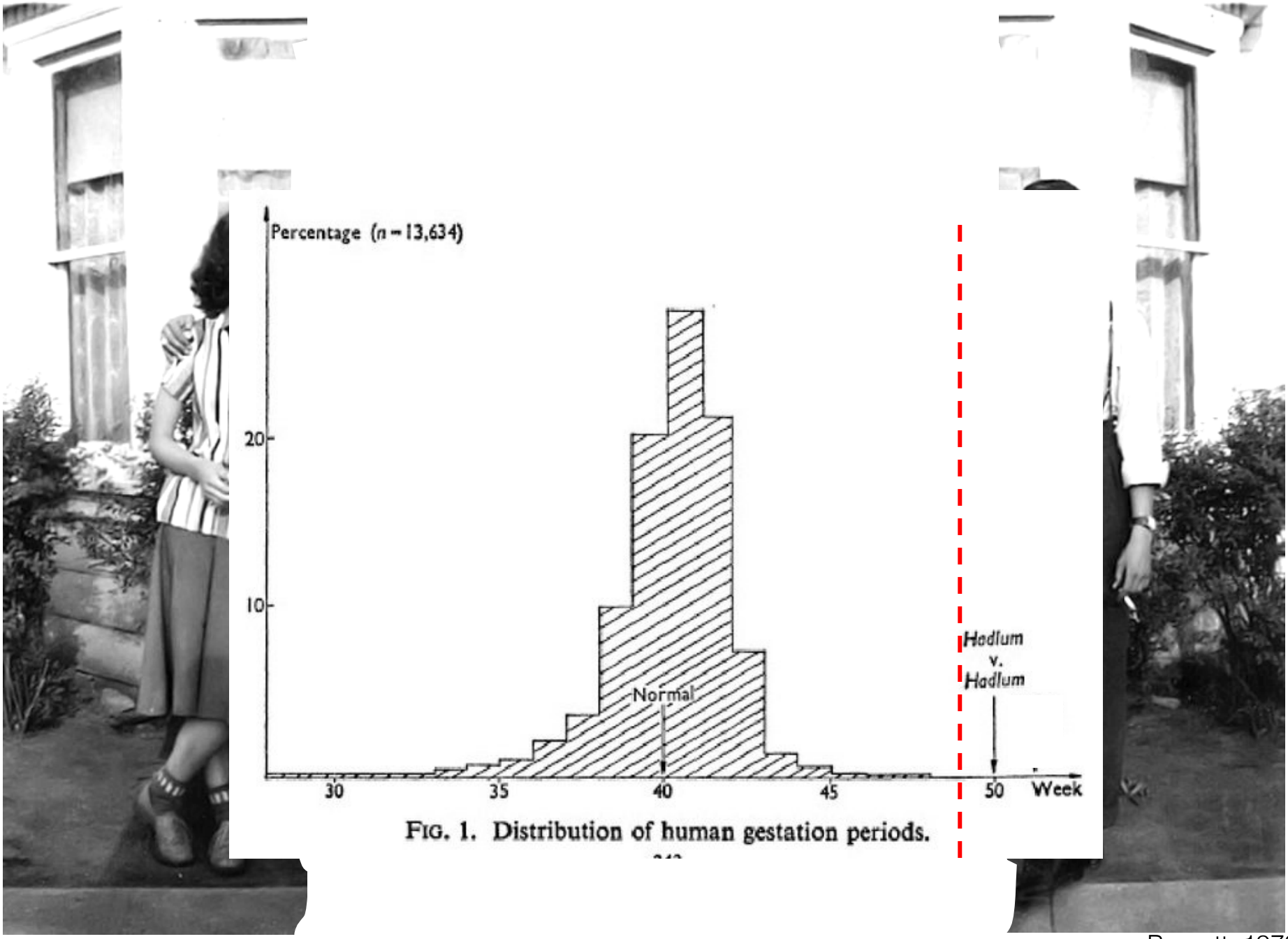

FIG. 1. Distribution of human gestation periods.

Barnett, 1978

thank you!

thoughts?

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or find me at www.jendannals.com