Yes-No Sieve for Logical Operators

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Abstract

Truth tables (Post, 1921; Wittgenstein, 1922) classify 16 logical operators between two yes-no propositions (e.g., 'p' and 'q') that produce offspring yes-no ('r') values. A Boolean algebra sieve allows for detection of the dominant logical operator, if present, within apparently random sets of yes and no responses. Whereas 100 randomized binary values return 43% to 58% positive identification, a logical operator-generated list is 100% compatible with the operator or 0% compatible with its complement. This sieve can guide research design refinements by probabilistically identifying logical relations between variables when initial data contains only presence-absence signal for p, q, and r.

Rationale

Truth tables (Post, 1921; Wittgenstein, 1922) allow for classification of 16 logical operators between two propositions (e.g., p and q) with all combinations of true and false values that produce in turn all possible true or false offspring proposition (r) values. Detecting what relation (logical operator) two 'states of affairs' have to the occurrence of another can be done quite practically using a Boolean sieve. Simulation work allows for perfect detection and recovery of otherwise apparently random sets of yes and no responses with identical expected proportions among several competitors. Specifically, a random list of 100 values of 1 or 0 with equal likelihood returns a proportion between 43% and 58% when sifted through the algebraic binary sieve. When a specific operator is used to generate a list of 100 values of 1 or 0, perfect identification can be had either by a 100% match to the operator values or 0% match to its complement. This procedure has been instantiated with operator E ('Neither or Both') and its perfect complement, operator J ('Only One'). This contrasts with continued stochastic drift around 50% for its fair competitors, each also having a 50% expectancy, namely the complementary operators G and H ('Not q', 'q'), and F and I ('Not p', 'p'). In the example, the logical relation between proposition p and proposition q with result r is identified as operator E, such that occurrence r is the case either when both p and q are not the case, or when p and q are both the case. Across the 16 operators in the Truth Table paradigm, listed by operator letter convention and practical descriptive, one complement is suited to 100%-0% expectancy: V – O ('Always', 'Never'), four are applicable to a 75%-25% expectancy: A – X ('All but Both', 'Only Both'); C – L ('All but p only', 'Only p'); B – M ('All but q only', 'Only q'); D – K ('All but Neither', 'Only None') and three are applicable to a 50%-50% expectancy in the dataset: G – H ('p', 'Not p'); F – I ('q', 'Not q'), and E – J ('Neither or Both', 'Only One'). This type of logical operator sieve can be a guide to inform more involved research design when initial data contains only clear presence-absence signal for states of affairs p and q and occurrence r.

Introduction

The premise of this work is charting the relation between three 'states of affairs', or truth conditions A, B, and C. Specifically, if truth states "A" and "B" can be ascertained and indexed to truth state "C", then the necessary, partial, or lack of relation between A and B in relation to the state of C can be evaluated in minute triangulations, especially as feasible in large data sets to examine for a consistent relation of A and B that is indicative of what C might be.

In speaking of truth states, truth tables are an exhaustive formulation of the possible relations between A, B, and their (posited) result, C. Truth tables as typically arranged are depicted in Table 1, below. A charting 'positive space' between the four possible points in a cube defined by corners (x, y, z) corresponding to (0, 0, 0) and (1, 1, 1) can be used to depict the endpoint truth values of A,B, and C, together with interim values along a plausible curved surface, as per the 'product equation' columns in Table 1. What is potentially useful about these surfaces is that they preserve a single equational definition to a continuous surface that links the four possible truth table endpoints.

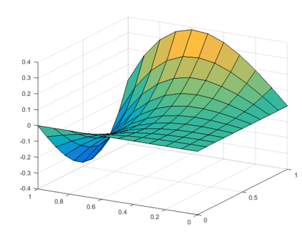
Table 1

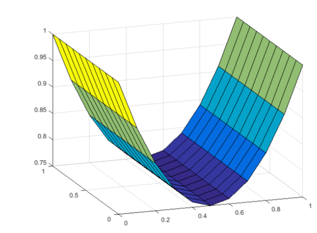
Characteristics of the Sixteen Logical Operators arranged by Eight Complementary Pairs

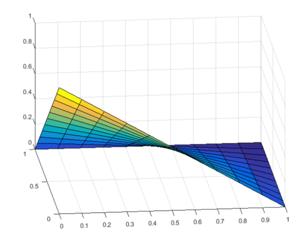
Operation	Complement Operation	*Operator*	*Complement Operator*	Operation result r*	Complement result r*	Common referents for complements		Product Surface Description	Product Surface Description – Complement	Verbal Logic for unit r ('true')	Verbal Logic for unit r ('true') – Complement	Positive Space Product Equation	Positive Space Product Equation - Complement
Logical disjunction	Logical NOR	**A**pq	**X**pq	T T T F	F F F T	OR	NOR	The Collector; low point front left, three high points	The Protector; High point front left, three low points.	'All but Neither'	'Only None'	$(x-y)*(x-y)+x*y$	$(1-x)*(1-y)$
Converse implication	Converse nonimplication	**B**pq	**M**pq	T T F T	F F T F	p ← q	p <-/- q	Manta Ray Ascending; low point back left, three high points along two high edges	Tarp Sagging; high point back left, three low points with one edge front right, one sag back right	'All but q only'	'Only q'	$x+(1-x)*(1-y)$	$y*(y-x)$
Material implication	Material nonimplication	**C**pq	**L**pq	T F T T	F T F F	p → q	p -/-> q	Downward Draping; low point front right, three high points, sag back left, edge front left	Tarp Sagging; high point front right, three low points, back left sag, front left edge	'All but p only'	'Only p'	$(1-x)*(1-x)+x*y$	$x*(x-y)$
Logical NAND	Logical conjunction	**D**pq	**K**pq	F T T T	T F F F	NAND	AND	The Collector, low point back left, three high points	The Protector; high point back right, three low points, two edges	'All but Both'	'Only Both'	$(1-x)*(1-y)+(y-x)*(y-x)$	$x*y$
Logical biconditional	Exclusive Disjunction	**E**pq	**J**pq	T F F T	F T T F	XNOR	XOR	The Saddle or 'Pringle'; diagonal high points, back left and front right, two low points opposite diagonal	Manta Ray Horizontal; diagonal high points, back left and front right, diagonal lowline between lows	'Neither or Both'	'Only One'	$(1-x)*(1-y)+x*y$	$(x-y)*(x-y)$
Negation of p	Projection of p	**F**pq	**I**pq	F F T T	T T F F	NOT p	p	Brighter Ski Jump; low edge right, high edge left	The Ski Jump; low edge left, high edge right	Not p	p	$(1-x)*(1-x)$	$x*x$
Negation of q	Projection of q	**G**pq	**H**pq	F T F T	T F T F	NOT q	q	Brighter Ski Jump; low edge left, high edge right	The Ski Jump; low edge right, high edge left	Not q	q	$(1-y)*(1-y)$	$y*y$
Contradiction	Tautology	**O**pq	**V**pq	F F F F	T T T T	False	True	Twisted Noodle; four low points, countervailing back left sag and back right flare, two front edges	Parabolic Half-Pipe; four high points, parallel edges high left and right, with parallel trough in center	None	All	$x*y*(x+y)*(x-y)$	$x+(x-1)(x-1)$, $y+(y-1)(y-1)$

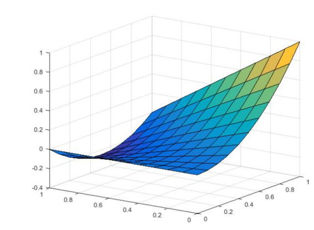
Method

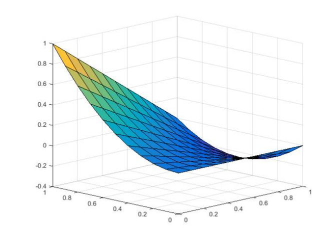
Figures below depict a plausible, single-equation surface connecting the 'four pillars' (truth table pairing of A and B values: TT,TF,FT, FF) with the value for C, the 'height' value. Values of A, B, and C, correspond to *p*, *q*, and *r* of Truth Table notation. In the figures below, A is the 'x axis' (viewer's right). B is shown on the horizontal axis perpendicular to the x axis (viewer's left). C is the 'height' axis (far left). The origin (0,0,0) is in the foreground for comparison. An interval of .1 is used for the gridlines (MATLAB 3-D plot function). Figures are arranged in 'families' by row. Row 1 figures have four identical heights, Row 2, three low points, Row 3, three high, Row 4, diagonal high points, Rows 5 and 6, single-edge high points.

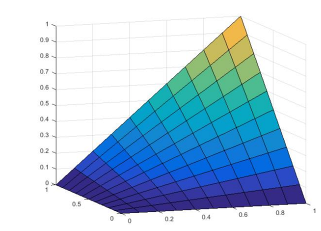

Operator O: None

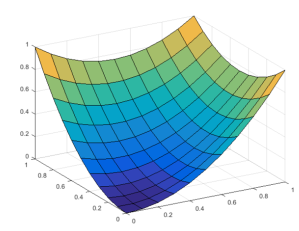

Operator V: All

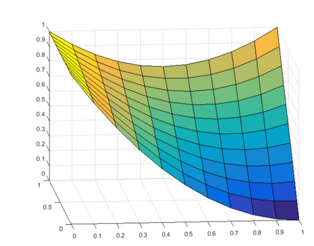

Operator X: Only Neither

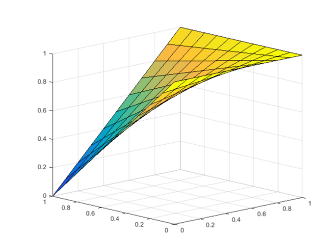

Operator L: Only A

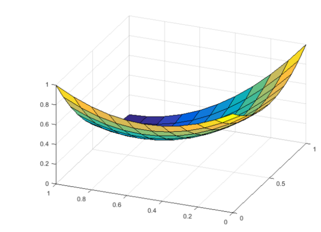

Operator M: Only B

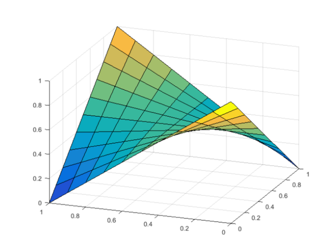

Operator K: Only Both

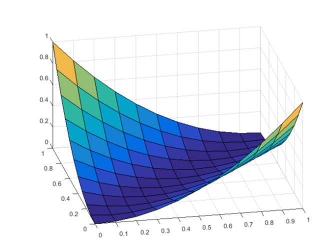

Operator A: All but Neither

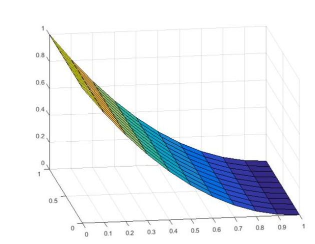

Operator C: All but A only

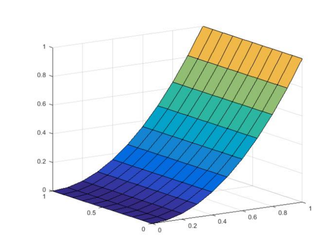

Operator B: All but B only

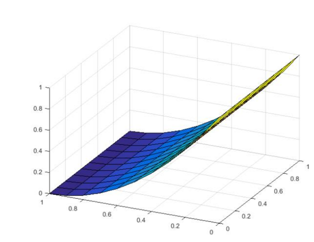

Operator D: All but Both

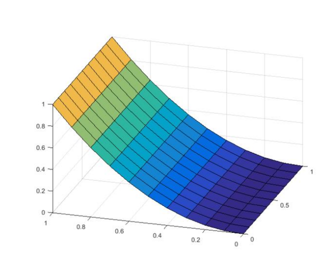

Operator E: Neither or Both


Operator J: Only One


Operator F: Not A


Operator I: A


Operator G: Not B


Operator H: B

Simulation Results

The system of equations that depicts the product surfaces (some kind of multiplication) can also be used as a filter for presence absence pattern. Each equation generates a unique and constant four digit binary signature in a two by two configuration for the C value based on the four permutations of dichotomous values of A and B. As such, a repeated pattern of occurrence *of the particular logical relation* can be detected uniformly *across all four possible manifestations of the arrangement of A and B values.*

A simulation was undertaken to study the sensitivity of the system of logical operator equations to logical operators in three data sets constructed with pure, random, and mixed generation of the large binary sample from a specific equation, from a random seed, and from a combination of the two. The pure sample showed a perfect match (100%) with the operator that generated it, while retaining a 50-50 distribution of 'true' and 'false' values. The random sample matched evenly with all 50%-50% operators, ranging from 43% to 58%. A confirming result was the perfect non-match (0%) with the complement operator to the generating operator, suggesting that only eight equations are needed with perfect complementarity. Similar tests showed that the same phenomena occurred, within stochastic variance of random procedures, for the 25-75, and 0-100 distributions of true and false values.

Potential Applications

Decision-making for the design and interpretation of experiments is central to scientific research. The soft sciences (Psychology, Sociology, Economics) often use distributions, ranges of errors, and correlational work to reflect the 'soft' nature of the latent constructs studied. By contrast, hard sciences (Physics, Chemistry, Engineering), although they access ranges of measurement error, often benefit from the assumption of an immutable, tangible quantity as the target of measurement (How much force? What is the chemical content? What are the logical components of a system). The approach delineated here uses the concept of experimental space used for example in Engineering (points on a binary cube) for defining the range of values necessary to chart the interaction of three variables (i.e., A, B, and C are (x,y,z), with an origin at (0,0,0) and a furthest point at (1,1,1)) (e.g. see Box, Hunter, & Hunter, 1978).

The addition with the present approach is the feasibility of examining *in-between spaces with an integrated a priori set of expectations* (cf. decisional control model in Shanahan & Neufeld, 2010), generated by the system of equations, that is particularly applicable where there is an interaction between the two independent variables (A and B, as x and y; see e.g., Kirk, 1982). Such spaces may be propitious to examine when value for A and B are more easily obtained as proportions or probabilities (between 0.0 and 1.0). As such, a correlation value for A and a probability for B could be used to track **the** likelihood of exceeding**ing** some pre-determined threshold for C, such as .50, .60. or .95, especially where A and B may interact. Areas in indigo and dark blue on the set of figures (center panel) reflect a likelihood of C clearly below .50; areas in orange, and yellow reflect a likelihood above .50. Areas in green, teal, light blue reflect a likelihood of C in the intermediate range (centered around .50).

Conclusion

The ability to identify zones of likely experimental effect is valuable in allocating research resources. If a fundamental relation between independent binary predictors A and B with dependent observed variable C can be established by looking at previous data, targeted experimentation can examine other points on the continuous experimental surface linking all four logic 'posts', the standard four permutation of two binary variables (T T, T F, F T, F F).

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