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New paradoxes in intertemporal choice

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Abstract

Similar to research on risky choice, the traditional analysis of intertemporal choice takes the view that an individual behaves so as to maximize the discounted sum of all future utilities. The well-known Allais paradox contradicts the fundamental postulates of maximizing the expected value or utility of a risky option. We describe a violation of the law of diminishing marginal utility as well as an intertemporal version of the Allais paradox.

Keywords: intertemporal choice, risky choice, cancellation.

1 Introduction

In the field of intertemporal choice, the discounted-utility (DU) theory proposed by Paul Samuelson in 1937 was presented not only as a valid normative standard but also as a descriptive theory of actual intertemporal choice behavior (Frederick, Loewenstein, & O'Donoghue, 2002; Samuelson, 1937). In its general form, the DU theory proposes that the value of an option, (x; t), is the product of its present utility, U(x), and an exponential temporal discounting function, F(t), where t is the time at which x is acquired. The overall value of a mixed option, $A = \{(x_1, t_1), (x_2, t_2), \ldots\}$, denoted $V(A)$, is simply the sum of these products. That is, $V(A) = \Sigma$ U(x_i) F(t_i). An option A will be preferred to an option B if and only if $V(A) > V(B)$. However, a large body of empirical evidence demonstrates that people systematically violate this theory. This includes the common difference effect, the magnitude effect, the gain-loss asymmetry, the delay-speedup asymmetry, and so on (Benzion, Rapoport, & Yagil, 1989; Prelec & Loewenstein, 1991; Thaler, 1981). This situation has led researchers to consider extensions and modifications of the DU theory to reconcile it with the experimental data.

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This paper is dedicated to Prof. Maurice Allais in memory of the historical importance of the Allais paradox.

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The most prominent idea to account for these anomalies is the hyperbolic discounting model (Ainslie, 1975). This model suggests that the discount rate is not dynamically consistent but that the rate is higher between the present and near future and lower between the near and far distant future. Numerous theories have been developed by transforming the discount function to other forms, from one-parameter hyperbolic discounting (Mazur, 1984) to generalized hyperbolic discounting (Loewenstein & Prelec, 1992), to proportional discounting (Harvey, 1994), and to quasi-hyperbolic discounting (Laibson, 1997). However, these models focus on intertemporal choice between pairs of single-dated outcomes represented as pure gains or losses. When these models are applied to intertemporal choice between pairs of multiple-dated outcomes in mixed contexts, there is general agreement on the additive assumption and the independence assumption. With an apt transformation of the discounting rate, the additive assumption means that preferences for outcome sequences are based on a simple aggregation of their individual components within intertemporal choice (Loewenstein & Prelec, 1993). The independence assumption means that the value or utility of an outcome in one period is independent of outcomes in other periods (Prelec & Loewenstein, 1991).

Because risk and delay might be psychologically equivalent, or at least analogous, and because similar psychological processes might underlie risk and intertemporal choice (Weber & Chapman, 2005), theoretical development in intertemporal choice has progressed steadily along a similar route as that of risky choice (Loewenstein & Prelec, 1992). Both lines of research have spawned a large number of variant models. Although the functional forms differ, most theories assume a maximization principle; that is, people calculate the mathematical expectation of each outcome and add them together before choosing the option that maximizes overall value or utility. A minor difference is that the existing models of in-

Table 1: An illustration of the multiple-dated outcomes problems.

	Options	Time 1	Time 2	Time 3
Problem I	A	a	0	0
	B	0	0	b
Problem I′	C	a	X	0
	D	0	X	b
Problem II	A	a	0	0
	B	0	0	b
Problem II′	C	a+X	0	0
	D	X	0	b

tertemporal choice are relatively underdeveloped and are less flexible in dealing with empirical challenges. For example, research on risky decision making does not treat risky choice as limited to pure gains or pure losses but has been extended to include mixed outcomes involving both gains and losses. Examples include the sign-dependent utility model (Einhorn & Hogarth, 1986), the rank- and sign-dependent utility model (Luce & Fishburn, 1991), and the transfer of attention exchange model (Birnbaum, 2008).

The well-known Allais paradox (Allais, 1953) contradicts the fundamental postulates of maximizing the expected utility of a risky option. The paradox presents a violation of the cancellation axiom, which asserts that, if two options have a common consequence under a particular event, the preference order of the options should be independent of the value of that consequence (Savage, 1954). Since then, many new descriptive theories of risky choice have abandoned the maximization assumption (e.g., Brandstätter, Gigerenzer, & Hertwig, 2006; Li, 2004; Rubinstein, 1988).

Most models of intertemporal choice have not yet abandoned the additive assumption and the independence assumption. These two assumptions would lead to the cancellation axiom, which indicates that a preference between two sequences with elements in common does not depend on the nature of the common elements. Table 1 illustrates an example of the multiple-dated outcomes problem, which would be used to test the cancellation axiom. In Problem I, the additive models predict that adding a common element X at Time 2 to both option A and option B would not change the preference orderings. The violation of cancellation would be observed if the preference orderings were different between Problem I and Problem I′. However, if Allais's proposition applies to intertemporal choice, we will eventually encounter an intertemporal version of the Allais paradox. We first illustrate our point with a paradox that is an intertemporal-type violation of the cancellation axiom.[1]

2 Experiment 1: Violation of the cancellation axiom

2.1 Method

The present experiment used intertemporal choices that were composed of multiple outcomes. An option O [x_1, t_1; x_2, t_2;. . . ; x_n, t_n] is a contract that yields outcome x_i with a delay of t_i units of time. We constructed pairs of temporal choices, as illustrated in Table 2. For example, option A offers ¥1,000,000 now and yields a loss of ¥2,000,000 in 1 year.

The participants were informed that there was no "correct" answer and that the aim of the study was to find out how people made decisions. They were asked to imagine that the choices were real and to indicate the option they preferred in such cases. Each participant was paid ¥5 for participation.

Ninety undergraduates from Huazhong University of Science and Technology (70 males and 20 females) with no special training in decision theory were asked to consider the following two problems.

Problem 1 Imagine that you had to choose between the following two options:

A: ¥1,000,000 now and ¥−2,000,000 in 1 year

B: ¥−2,000,000 in 1 year and ¥5,000,000 in 10 years

Problem 2 Imagine that you had to choose between the following two options:

C: ¥1,000,000 now

D: ¥5,000,000 in 10 years

Ninety-three postgraduates (46 males and 47 females) from Peking University and the Graduate University of the Chinese Academy of Sciences were asked to consider the following two problems.

Problem 3 Imagine that you had to choose between the following two options:

A: Gain 5 apples now and lose 6 apples tomorrow

B: Lose 6 apples tomorrow and gain 8 apples in 1 week

Problem 4 Imagine that you had to choose between the following two options:

C: Gain 5 apples now

D: Gain 8 apples in 1 week

[1]The idea to remove the components that are explicitly common to the alternatives was developed in the spring of 2008 when the authors observed crabapple flowers falling from branches with the passage of time.

Table 2: Percentages of choices of problems in Experiments 1–3

	Problem	Options	N (%)
Experiment 1	1	A: ¥1,000,000 now and ¥−2,000,000 in 1 year	26 (28.9%)
		B: ¥−2,000,000 in 1 year and ¥5,000,000 in 10 years	64 (71.1%)
	2	C: ¥1,000,000 now	69 (76.7%)
		D: ¥5,000,000 in 10 years	21 (23.3%)
	3	A: Gain 5 apples now and lose 6 apples tomorrow	15 (16.1%)
		B: Lose 6 apples tomorrow and gain 8 apples in 1 week	78 (83.9%)
	4	C: Gain 5 apples now	61 (65.6%)
		D: Gain 8 apples in 1 week	32 (34.4%)
Experiment 2	5	A: ¥10,000 now	30 (33.3%)
		B: ¥30,000 in 1 year	60 (66.7%)
	6	C: ¥10,010,000 now	46 (51.1%)
		D: ¥10,000,000 now and ¥30,000 in 1 year	44 (48.9%)
	7	A: ¥1 now	17 (18.3%)
		B: ¥3 the day after tomorrow	76 (81.7%)
	8	C: ¥100, 001 now	46 (49.5%)
		D: ¥100,000 now and ¥3 the day after tomorrow	47 (50.5%)
	9	A: ¥100 now	31 (20.7%)
		B: ¥200 in 1 month	119 (79.3%)
	10	C: ¥100,000,000,100 now	72 (48.0%)
		D: ¥100,000,000,000 now and ¥200 in 1 month	78 (52.0%)
	11	A: ¥1 now	44 (29.5%)
		B: ¥3 the day after tomorrow	105 (70.5%)
	12	C: ¥100,000,000,001 now	77 (51.7%)
		D: ¥100,000,000,000 now and ¥3 the day after tomorrow	72 (48.3%)
Experiment 3	13	A: ¥1,000,000 now	85 (72.0%)
		B: ¥5,000,000 in 10 years	33 (28.0%)
	14	C: ¥1,000,000 now and ¥6,000,000 in 1 year	57 (48.3%)
		D: ¥6,000,000 in 1 year and ¥5,000,000 in 10 years	61 (51.7%)
	15	E: ¥1,000,000 now and ¥−2,000,000 in 11 years	59 (50.0%)
		F: ¥5,000,000 in 10 years and ¥−2,000,000 in 11 years	59 (50.0%)

2.2 Results

Table 2 presents the results of Experiment 1. The number of respondents who answered each problem is denoted by N, and the percentage of those who chose each option is given in brackets. The data show that 71.1 percent of the participants chose B in Problem 1, and 76.7 percent of the participants chose C in Problem 2. A McNemar test revealed that a significantly greater number of participants chose option B in Problem 1 compared with those who chose option C in Problem 2 ($p < 0.001$).

This pattern of preferences violates any existing discounted utility theory that assumes the cancellation axiom. Given that each component is added separately, the cancellation axiom implies that, if two options have a common outcome (the same outcome produced by the same event at the same time), the preference order induced by other components of the options will be independent of that outcome. Considering Problem 1 and Problem 2 with the cancellation axiom, with $u(0)=0$, the first preference implies

$$F(\text{now})\,u(1{,}000{,}000) + F(1\text{ year})\,u(-2{,}000{,}000) < F(1\text{ year})\,u(-2{,}000{,}000) + F(10\text{ years})\,u(5{,}000{,}000),$$

where < represents the strict preference of the individual.

Subtracting $F(1\text{ year})\,u(-2{,}000{,}000)$ from both sides, we have

$$F(\text{now})\,u(1{,}000{,}000) < F(10\text{ years})\,u(5{,}000{,}000),$$

while the second preference implies the reverse inequality:

$$F(\text{now})\,u(1{,}000{,}000) > F(10\text{ years})\,u(5{,}000{,}000).$$

Note that Problem 2 is obtained from Problem 1 by removing "¥−2,000,000 in 1 year" from both options. This pattern of preference contradicts the cancellation axiom. The choice between options A (1,000,000 now; −2,000,000 in 1 year) and B (−2,000,000 in 1 year; 5,000,000 in 10 years) cannot be easily reduced by the cancellation of the choice between options C (1,000,000 now) and D (5,000,000 in 10 years).

We observed the same pattern in Problems 3 and 4. Most participants (83.9%) chose to lose 6 apples first and gain 8 apples later in Problem 3, and most of the participants (65.6%) chose to gain 5 apples now in Problem 4 (see Table 2). Note that Problem 4 is obtained from Problem 3 by removing "lose 6 apples tomorrow" from both prospects under consideration. This result, together with the finding in monetary problems, points to a violation of the cancellation axiom.

3 Experiment 2: Violation of the law of diminishing marginal utility

Experiment 1 showed that the additivity and independence would lead to a violation of the cancellation axiom by adding a common element X at a different time from the original options. If the common element X was added at the same time as the original options (e.g., Problem II and Problem II′ in Table 1), the additive models would assume that outcome b at Time 3 in option B was the same as outcome b in option D. In Experiment 2, by adding a common element X at the same time as the original options, we demonstrated that the additivity and independence would also lead to a violation of the law of diminishing marginal utility, which states that the marginal utility of an extra dollar in payoffs declines with increases in income or wealth (Tversky, 1991; Wakker, Köbberling, & Schwieren, 2007).

3.1 Method

The procedure for Experiment 2 was the same as that for Experiment 1. Each participant was paid ¥5 for participation. Ninety undergraduates from Huazhong University of Science and Technology (70 males and 20 females) with no special training in decision theory were asked to consider the following two problems.

Problem 5 Imagine that you had to choose between the following two options:
A: ¥10,000 now
B: ¥30,000 in 1 year

Problem 6 Imagine that you had to choose between the following two options:
C: ¥10,010,000 now
D: ¥10,000,000 now and ¥30,000 in 1 year

Ninety-three postgraduates (46 males and 47 females) from Peking University and Graduate University of Chinese Academy of Sciences were asked to consider the following four problems.

Problem 7 Imagine that you had to choose between the following two options:
A: ¥1 now
B: ¥3 the day after tomorrow

Problem 8 Imagine that you had to choose between the following two options:
C: ¥100, 001 now
D: ¥100,000 now and ¥3 the day after tomorrow

One hundred and fifty Beijing Forestry University students (57 males, 91 females, and 2 unknown) were asked to consider the following four problems.

Problem 9 Imagine that you had to choose between the following two options:
A: ¥100 now
B: ¥200 in 1 month

Problem 10 Imagine that you had to choose between the following two options:
C: ¥100,000,000,100 now
D: ¥100,000,000,000 now and ¥200 in 1 month

Problem 11 Imagine that you had to choose between the following two options:
A: ¥1 now
B: ¥3 the day after tomorrow

Problem 12 Imagine that you had to choose between the following two options:
C: ¥100,000,000,001 now
D: ¥100,000,000,000 now and ¥3 the day after tomorrow

3.2 Results

Table 2 also presents the results of Experiment 2. The data show that 66.7 percent of the participants chose option B in Problem 5, and 51.1 percent of the participants chose option C in Problem 6. While option B was strongly preferred to option A, option C was weakly preferred to option D. A McNemar test revealed a significant increase in the number of participants who chose the immediate options, from 33.3 percent choosing option A in Problem 5 to 51.1 percent choosing option C in Problem 6 (p = 0.017). Based on the additive models of intertemporal choice, the first preference implies

$F(\text{now})\, u(10{,}000) < F(1\text{ year})\, u(30{,}000)$,

while the second preference implies

$F(\text{now})\, u(10{,}010{,}000) \approx F(\text{now})\, u(10{,}000{,}000) + F(1\text{ year})\, u(30{,}000)$

or

$F(1\text{ year})\, u(30{,}000) \approx F(\text{now})\, u(10{,}010{,}000) - F(\text{now})\, u(10{,}000{,}000)$.

Taken together,

$F(\text{now})\, u(10{,}000) < F(1\text{ year})\, u(30{,}000) \approx F(\text{now})\, u(10{,}010{,}000) - F(\text{now})\, u(10{,}000{,}000)$.

This pattern of preferences violates the law of diminishing marginal utility. Note that options C and D are easily obtained from options A and B by the insertion of the common ¥10,000,000 now. However, this common outcome caused a preference to shift to option C.

McNemar tests revealed another significant increase in the number of participants who chose the immediate options, from 18.3 percent choosing option A in Problem 7 to 49.5 percent choosing option C in Problem 8 ($p <$ 0.001); there were also significant increases ($p <$ 0.001 for all) in the numbers of participants who chose the immediate options between option A in Problem 9 (29.5%) and option C in Problem 10 (51.7%), and between option A in Problem 11 (20.7%) and option C in Problem 12 (48.0%). Given that we used different samples and outcomes, it is relatively safe to assume that the violation of the law of diminishing marginal utility is reliable and stable.

4 Experiment 3

To identify the psychological mechanism of the intertemporal choice, Loewenstein and Prelec (1993) proposed a model for preferences over outcome sequences that involved abandoning the additive assumption.[2] According to Loewenstein and Prelec's (1993) model for preferences over outcome sequences, people typically favor sequences that improve over time. In that case, options A and B in Problem 1 can be framed as a decreasing and increasing sequence, respectively, so that preference for B over A increases. To test whether this model can account for our observed paradox, we conducted Experiment 3.

4.1 Method

The procedure was the same as that in Experiments 1 and 2. Each participant was paid ¥5 for participation. One hundred and eighteen students from the Graduate University of the Chinese Academy of Sciences and the Central University of Finance and Economics were asked to make the following three pairs of choices.

Problem 13 Imagine that you had to choose between the following two options:
A: ¥1,000,000 now
B: ¥5,000,000 in 10 years

Problem 14 Imagine that you had to choose between the following two options:
C: ¥1,000,000 now and ¥6,000,000 in 1 year
D: ¥6,000,000 in 1 year and ¥5,000,000 in 10 years

Problem 15 Imagine that you had to choose between the following two options:
E: ¥1,000,000 now and ¥−2,000,000 in 11 years
F: ¥5,000,000 in 10 years and ¥−2,000,000 in 11 years

In Problem 14, as the common outcome makes option C an increasing sequence and option D a decreasing sequence, preference for C over D might increase according to Loewenstein & Prelec's (1993) model. In Problem 15, because both options are decreasing sequences, Loewenstein & Prelec's (1993) model predicts that the tendency to choose E over F should be similar to the tendency to choose A in Problem 13.

4.2 Results

The result showed that 72.0 percent of the participants chose A in Problem 13, 51.7 percent of the participants chose D in Problem 14, and 50.0 percent chose F in Problem 15 (Table 2). The McNemar t-tests revealed that the effects of common outcome were significant (p <.001 for all), indicating that our paradox survives in these increasing/decreasing settings.

Taken together, these results suggest that Loewenstein and Prelec's (1993) model might partially explain our findings in Experiment 1 but might not provide a satisfactory explanation for the result of Experiment 3.

5 Discussion

In contrast to many previous violations of the DU theory, which have been used to modify the functions of DU the-

[2] We are grateful to an anonymous referee and Editor Jonathan Baron for drawing our attention to Loewenstein and Prelec's (1993) work.

ories, the present findings challenge the explanation and prediction of the standard constant discount utility functions and the hyperbolic discounting theories that assume the cancellation axiom. We do not doubt the possibility that some future revamped discounted utility theories could accommodate our data, as each option chosen can be taken as evidence that the decision maker is still calculating some form of mathematical expectation.

When the Allais paradox questioned the cancellation assumption in risky decision making, maximization principle proponents argued that any definite rule for choosing between risky prospects can be described as a maximization of some function. The issue is not whether choice can be described as a maximization but which function is being maximized (for a more detailed argument, see Li, 1996). In the same vein, it would not be surprising if those who remain devoted to expectation-maximizing in intertemporal choice argue that our findings reject the existing discount-utility functions but cannot invalidate the utility maximization algorithm. Indeed, as Carlin said, "Attacking the maximization principle is akin to the classical Greek story of slaying the hydra ... for each head one cuts off, two grow in its stead" (Carlin, 1996).

Rather than adopting a general functional form to reconcile the experimental data, deriving certain fundamental psychological properties from the natural decision-making process might be a more promising strategy for constructing a theory of intertemporal choice. With its focus on the case of hyperbolic discounting, the methodology of "economics and psychology" has been questioned by theoretical economists (Rubinstein, 2003). Just as Rubinstein (2003) suggested, "Combining 'economics and psychology' requires opening the black box of decision makers instead of modifying functional forms."

By utilizing the notions of utility improvement and uniformness with respect to global rather than local sequence properties, Loewenstein and Prelec's (1993) proposed model goes beyond previous attempts to account for the intertemporal choice. However, their model cannot accommodate all of the violation of the cancellation axiom. To open the black box of intertemporal choice, other changes to developing new models may be required.

One way to address this issue might be to take the wealth effect into account, which assumes the non-independence axiom. Wealth effects mean that "in addition to the consumption stream the utility function is also sensitive to the per capita capital stock of the society" (Kurz, 1968). That is, the utility of an outcome depends on what has happened before. Take Problem 1 as an example; the immediate outcome of ¥1 million would affect the utility of ¥−2,000,000 in option A, whereas the immediate outcome of ¥0 might affect the utility of ¥−2,000,000 in option B in a different way. Because the component of u(−2,000,000) actually represents different marginal utilities in the two options, we cannot subtract F(1 year) textitu(−2,000,000) from both sides of the equation. By suggesting the non-equivalence of the common outcome between the two options, the wealth effects account for the observed violations in Problems 1 and 2. However, the same logic would have a hard time explaining apple problems (Problems 3 and 4) because apples are non-durable goods and have nothing to do with wealth.

From a non-compensatory perspective, a number of attribute-based models might offer alternative explanations for the observed violations. These include the similarity-induced time preferences model (Rubinstein, 2003), the equate-to-differentiate model (Li, 2004), and the tradeoff model (Scholten & Read, 2010). The similarity-induced account assumes that an individual "uses a procedure that aims at simplifying the choice by applying similarity relations" (Rubinstein, 2003). When comparing the choice between "¥1,000,000 now" and "¥5,000,000 in 10 years" in Problem 2, most people consider the money amounts to be similar; this is not the case for the time periods. Thus, the time dimension is the decisive factor. From an equate-to-differentiate point of view, human choice behavior can be viewed as a process in which people seek to equate a less significant difference between alternatives on one dimension (either *amount of payment* or *time of payment*), thus leaving the greater one-dimensional difference as the determinant of the final choice. The observed immediacy effect in Problem 2 can thus be viewed by the equate-to-differentiate account as a decision in which people seek to equate the less significant difference between the options on the "*amount of payment*" dimension (e.g., ¥1,000,000 vs. ¥5,000,000), thus leaving the greater difference between the options on the "*time of payment*" dimension (e.g., now vs. in 10 years) to be differentiated as the determinant of the final choice.

Indeed, when facing the choice between A (¥1,000,000 now; ¥−2,000,000 in 1 year) and B (¥−2,000,000 in 1 year; ¥5,000,000 in 10 years) or between C (¥10,010,000 now) and D (¥10,000,000 now; ¥30,000 in 1 year), the account for the modal preferences in these problems is less clear cut, including those provided by the similarity-induced time preferences and the equate-to-differentiate model. The recently proposed tradeoff model (Scholten & Read, 2010) goes a step further and proposes that options are directly compared along the outcome and time attributes such that the option favored by the intra-attribute comparisons is chosen. In the choice between C and D in Problem 2, the time difference apparently outweighs the money difference such that C is preferred to D. In the choice between A and B in Problem 1, different people may do different things. Some may treat the problem as a choice between C and D, thus canceling the common element and leading to a preference for A over B. Others may engage in an intra-attribute comparison process by comparing only the difference between 1,000,000 and 0 with the difference between 5,000,000 and 0, thus leading to the preference for B over A. Thus, non-compensatory models might be required to clarify how to make intra-attribute comparisons when there are multiple outcomes in one option.

A more fundamental explanation for the change in preference between Problems 1 and 2 (and all other problems of this form) could be accounted for if we assume a strong preference for immediacy in Problem 2 and a preference for a net gain in Problem 1. Furthermore, the change in preference between Problems 5 and 6 (and all other problems of this form) could be accounted for by a "peanuts effect", which refers to the finding that individuals behave differently when presented with small versus large gambles (Prelec & Loewenstein, 1991). The participants were quite impatient in Problems 5, 7, and 11 when the monetary amount was small, which is consistent with the participants in most other studies (e.g., Keren & Roelofsma, 1995; Read, Loewenstein, & Kalyanaraman, 1999). However, the added outcome in Problems 6, 8, and 12 was immediate and so large that making two options approximately indifferent.

Moreover, the most commonly used paradigm for constructing intertemporal choices has been with a "pure gain/loss" task. That is, participants are asked to choose between a smaller gain received sooner and a larger gain received later, or to choose between a smaller loss received sooner and a larger loss received later. With this paradigm in mind, the decisions of the ant and the grasshopper in Aesop's classic fable are presumably constructed to choose between an immediate and a delayed reward. The decisions are made by an indulgent grasshopper who "luxuriates during a warm summer's day" and a patient ant who "stores food for the upcoming winter" (McClure, Laibson, Loewenstein, & Cohen, 2004). The decision is as simple as choosing between $10 today and $11 tomorrow.

However, readers should notice that the essential decision for the ant and the grasshopper in Aesop's fable is not simply to choose between an immediate and a delayed reward but to choose between "luxuriate in summer and die of hunger in winter" and "toil in summer and enjoy corn and grain in winter". The options they face are good exemplars of the intertemporal choice with mixed outcomes in real-world settings. This choice can actually be organized into two categories: "earlier gain and later loss" (e.g., drug abuse or usurious loan) and "earlier loss and later gain" (e.g., education investment or oil drilling). Similar to the decision for the ant and the grasshopper in Aesop's fable, readers should notice that some choices in the present study had mixed outcomes (e.g., Problem 1). To our knowledge, unlike models of risky choice (e.g., prospect theory), no extant models of intertemporal choice have been developed to generate gain and loss functions separately. This might be the reason why existing models have a hard time explaining our findings with the mixed outcome paradigm.

The "pure gain/loss" paradigm might have been shaped by the idea of interest rate (e.g., Modigliani, 1986). That is, people behave as though any gain provides an opportunity to earn interest (or, equivalently, to pay off a debt and avoid paying interest), making the interest or avoided debt a monetary gain (Baron, 2008). Nevertheless, as suggested by the re-analysis of the fable, an accurate model of the intertemporal choice should include consideration of zero or negative outcomes in the offered options. We can see that even making explicit the hidden zero in each option would increase individuals' preferences for larger delayed rewards (Magen, Dweck, & Gross, 2008). Similar findings can be seen as a lever that moves existing models of intertemporal choice to conceive models that accommodate the features of options with mixed outcomes.

To accommodate the present paradoxes, new models are needed to capture and include the features of options with mixed outcomes in the real world. A candidate model might modify the weighting function or the utility function of existing utility maximization models by abandoning additivity and independence. Alternatively, another candidate model might abandon the utility maximization algorithm and extend the non-compensatory decision rules to the multiple outcomes settings.

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