One of the giants of mathematical psychology
One of our founding fathers

J-.Cl. Falmagne

University of California, Irvine

R. Duncan Luce, 1925-2012
One of the giants of mathematical psychology

Eight books, many of them best sellers.

(1957) Luce, R.D. and Raiffa, H. *Games and Decisions: Introduction and Critical Survey.* New York: Wiley. (Reprinted in 1989 by Dover Publications)

(1959) Luce, R.D. *Individual Choice Behavior: A Theoretical Analysis.* New York: Wiley.

(1971) Krantz, D.H, Luce, R.D., Suppes, P., & Tversky, A. *Foundations of Measurement, Vol. I*, Academic Press.

(1986) Luce, R.D. *Response Times.* New York: Oxford University Press.

(1989) (Suppes, P., Krantz, D.H., Luce, R.D., & Tversky, A. *Foundations of Measurement, Vol. II*, Academic Press.

(1990) Luce, R.D., Krantz, D.H., Suppes, P., & Tversky, A. *Foundations of Measurement, Vol. III*, Academic Press.

(1993) Luce, R.D. *Sound & Hearing*, Hillsdale, NJ: Erlbaum.

(2000) Luce, R.D. *Utility of Gains and Losses: Measurement-Theoretical and Experimental Approaches.* Mahwah, NJ: Lawrence Erlbaum Associates.

R. Duncan Luce, 1925-2012
One of the giants of mathematical psychology

- 11 co-edited volumes, especially the three volumes of the Handbook of Mathematical Psychology, published in 1963, which played a critical role in the foundation of our field.

- 250 papers published or in press (a couple of them).

R. Duncan Luce, 1925-2012
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Duncan Luce got his Ph.D. in mathematics from MIT in 1950.
But his interest in the behavioral sciences started earlier than that.
He published his first paper in 1949.

Luce, R.D.& Perry, A.D.
A method of matrix analysis
of group structure.
Psychometrika, 14, 95-116, 1949.

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He became famous quite early with his co-authored book:

> Luce, R.D. and Raiffa, H. *Games and Decisions: Introduction and Critical Survey.* New York: Wiley, 1957.
> (Reprinted in 1989 by Dover Publications)

It quickly became a classic work on game theory.

Mathematical psychology would have existed without Duncan Luce

But the chances are that it would have been a different discipline,

in style and content.

R. Duncan Luce, 1925-2012
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In 1959, he published what is perhaps the most influential book
in mathematical psychology.

- It was small.
- It was axiomatic, set-theoretical, theorem proving.
- Despite that, it was easy to read.
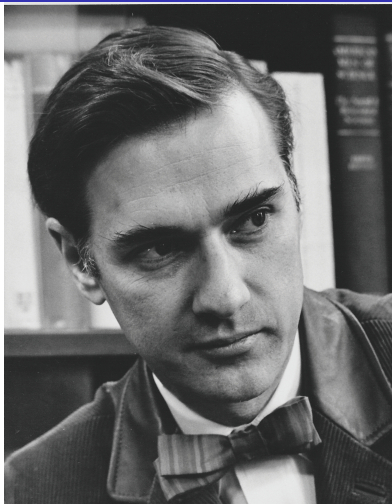- It was a revolution.

R. Duncan Luce, 1925-2012
One of the giants of mathematical psychology

Luce, R.D.
Individual Choice Behavior:
A Theoretical Analysis.
New York: Wiley, 1959.

In my view, the importance of this little red book was due to its style, rather than to its exact content.

Its effect was like reading, for the first time,
a couple of beautiful sonnets, and thinking:
'**maybe, I could write like that too!**.'

R. Duncan Luce, 1925-2012
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It set the stage for much of Duncan's later work, and in particular his reformalizing Fechner's psychophysics in modern mathematical lingo, in the form of **Fechner's Problem**[1].

A classical situation in psychophysics. The set A is a sensory continuum, such as a set of monochromatic visual stimuli varying only in intensity.

b a

With a and b in A and $a \leq b$, let $P(a, b)$ be the probability that b is judged brighter than a. So, we have, for some set $B \subseteq A \times A$,

$$P : B \to]0, 1[: ab \mapsto P(a, b).$$

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[1]Luce, R.D. & Galanter, E. *Discrimination*, a chapter in Volume I of the Handbook of Mathematical Psychology, R.D. Luce, R.R. Bush & E. Galanter, John Wiley, 1963.

Fechner's Problem amounts to finding a strictly increasing function $u : A \to \mathbb{R}_+$ and a strictly increasing function F such that

$$P(a, b) = F\big(u(b) - u(a)\big). \qquad (*)$$

Under which conditions on the function P do the functions u and F exist?

The difficulty is that

- if a is much smaller that b, then $P(a, b) = 1$

- and if a and b are very close, then $P(a, b) = 0$.

So, the interval B has to be defined appropriately if the function F is to be strictly increasing. In other words, we have an implicit '**restricted solvability**' condition of the $(*)$ equation here.

R. Duncan Luce, 1925-2012
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Fechner's Problem amounts to finding a strictly increasing function $u : A \to \mathbb{R}_+$ and a strictly increasing function F such that

$$P(a, b) = F\big(u(b) - u(a)\big). \qquad (*)$$

Under which conditions on the function P do the functions u and F exist?

Two things about this equation $(*)$:

- involves two unknown functions, which gives a hint that the theory of functional equations might be relevant;

- it also suggests a reformulation in the context of measurement theory.

In the framework and language of measurement theory, this problem can be rephrased as:

Fechner's Problem as a measurement theory problem.
- *Two sets A and $B \subset A \times A$.*
- *$\precsim$ a weak order on B (transitive, connected, reflexive).*

Find necessary and sufficient conditions on $(A, B, \precsim)$ such that, for some function $u : A \to \mathbb{R}$, we have

$$ab \precsim cd \quad \Longleftrightarrow \quad u(b) - u(a) \leq u(d) - u(c) \,.$$

The 'restricted solvability' condition is what made that problem tricky.

R. Duncan Luce, 1925-2012
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Many honors were granted to Duncan Luce through his life time.
What I give below is just a sample.

- 1963 Society of Experimental Psychologists
- 1966 American Academy of Arts and Sciences
- 1972 **National Academy of Sciences**
- 1976 Honorary degree, Harvard University
- 1980 Guggenheim Fellowship
- 1986 1986 American Association for the Advancement of Science Prize for Behavioral Science Research
- 1986 American Association for the Advancement of Science Prize for Behavioral Science Research
- 1994 UCI Distinguished Faculty Lectureship Award for Research
- 1994 American Philosophical Society
- 2003 **National Medal of Science** (presented by President Bush in 2005).
- 2007 **University of Waterloo, Honorary Doctorate of Mathematics**

R. Duncan Luce, 1925-2012
One of the giants of mathematical psychology

The years 1960-75 were exceptionally productive for Duncan.

I list a few high points. One of them was the continuation of his collaboration with Pat Suppes in the reshaping of Mathematical Psychology in the language of modern mathematics.

- 1964, foundation of the
  Journal of Mathematical Psychology,
  Editors: R. C. Atkinson, R. R. Bush,
  W. K. Estes, R. D. Luce, and P. Suppes.

- In 1964 he invited Janos Aczél, the world
  expert in functional equations theory, to
  spend the summer at Stanford. This was the
  beginning of a life long collaboration
  with Aczél, leading to many publications.

- 1971, publication of the first volume of
  Foundation of Measurement, with D. Krantz,
  P. Suppes & A. Tversky.

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- 1963-65, publication of the three volumes of the
  Handbook of Mathematical Psychology, and its two volumes of *Readings*
  By R. Duncan Luce, Robert R. Busch & Eugene Galanter.

R. Duncan Luce, 1925-2012
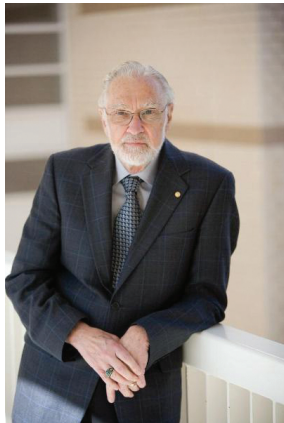## One of the giants of mathematical psychology

Also around that time, he started collaborating with David Green, of Signal Detection fame.

- Luce, R,D. and Green, D.M. The response ratio hypothesis for magnitude estimation. *JMP,* 15, 291-300. 1974.
- Green, D.M., and Luce, R,D. Counting and timing mechanisms in auditory discrimination and reaction time. In D.H. Krantz, R.C. Atkinson, R.D. Luce, and P. Suppes (Eds.) *Contemporary Developments in Mathematical Psychology, Vol. II.* San Francisco: Freeman. Pp. 372-415, 1974.
- Luce, R,D. , and Green, D.M. Ratios of magnitude estimates. In H.R. Moskowitz, B. Scharf, and J.C. Stevens (Eds.) *Sensation and Measurement.* Dordrecht, Holland: D. Reidel Pp. 99-111, 1974.
- Luce, R,D. , and Green, D.M.) Detection, discrimination, and recognition. In E.C. Carterette and M.P. Friedman (Eds.) *Handbook of Perception, Vol. II.* New York: Academic Press. Pp. 299-342. 1974.

R. Duncan Luce, 1925-2012
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Duncan Luce and Janos Aczél wrote many joint papers.

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Publications with Janos Aczél

- J. Aczél, R. D. Luce, G. Maksa. Solutions to Three Functional Equations Arising from Different Ways of Measuring Utility. *Journal of Mathematical Analysis and Applications*, vol. 204, no. 2, pp. 451- 471, 1996.

- J. Aczél, J.-Cl. Falmagne, and R.D. Luce. Functional equations in the behavorial sciences. *Japonica Mathematica*, 52(3):469–512, 2000.

- C. T. Ng, R. D. Luce, J. Aczél. Functional Characterizations of Basic Properties of Utility Representations. *Monatshefte Fur Mathematik*, vol. 135, no. 4, pp. 305-319, 2002.

- J. Aczél, R. Duncan Luce. Two functional equations preserving functional forms *Proceedings of The National Academy of Sciences*, vol. 99, no. 8, pp. 5212-5216, 2002.

- J. Aczél, R. D. Luce, Che Tat Ng. Functional equations arising in a theory of rankdependence and homogeneous joint receipts. Published in 2003.

- R. D. Luce, C. T. Ng, A. A. J. Marley, J.Aczél, Utility of gambling I. *Economic Theory*, vol. 36, no. 1, pp. 1-33, 2008.

- R. D. Luce, C. T. Ng, A. A. J. Marley, J. Aczél, Utility of gambling II. *Economic Theory*, vol. 36, no. 2, pp. 165-187, 2008.

R. Duncan Luce, 1925-2012
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You may ask: what is the point of this display of pages of titles of papers?

They show how, almost systematically, Duncan mathematized much of experimental psychology. His works included

- mathematical learning theory, with the Beta model
- choice theory
- psychophysics
- reaction times

in addition to his works in

— philosophy of science, with measurement theory, and
— economics, with choice theory and utility theory.

R. Duncan Luce, 1925-2012
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The number one collaborator of Duncan Luce in reframing
mathematical psychology in the language of modern mathematics
was certainly Patrick Suppes, another giant of mathematical psychology.
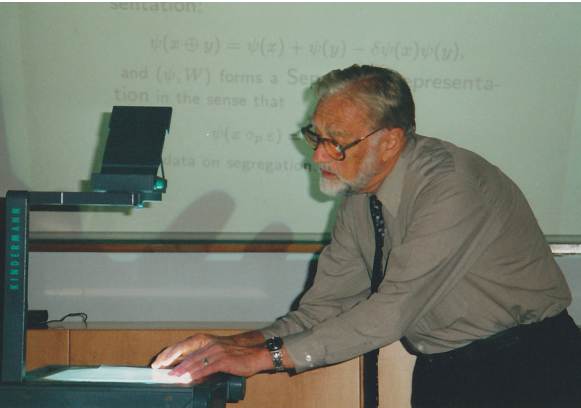
R. Duncan Luce, 1925-2012
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Both Luce and Suppes were instrumental in introducing the concepts, and methods of modern mathematics—based on axiomatization, set theoretical language and theorem proving—in mathematical psychology.

R. Duncan Luce, 1925-2012
One of the giants of mathematical psychology

They complemented each other ideally for the task at hand.

Duncan Luce was a professor of psychology and devoted most of his life to solving behavioral science related problems, while Patrick Suppes was a philosopher of science with a strong mathematical bent, interested in a very wide range of problems, and teaching in a philosophy department.

R. Duncan Luce, 1925-2012
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Some of Duncan's accomplishments in 1980-1990.

- Luce, R.D. *Response Times.*
  New York: Oxford University Press, 1986.

  And then the last two volumes
  on the Foundations of Measurement:

- Suppes, P., Krantz, D.H., Luce, R.D.,
  & Tversky, A. *Foundations of Measurement*,
  Vol. II, Academic Press, 1989.

- Luce, R.D., Krantz, D.H., Suppes, P.,
  & Tversky, A. *Foundations of Measurement,*
  Vol. III, Academic Press, 1990.

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- 1989-1990. Volumes II and III of *Foundations of Measurement*.

David Krantz

Duncan Luce

Amos Tversky

Pat Suppes

R. Duncan Luce, 1925-2012

One of the giants of mathematical psychology

We have seen that some of Duncan's work in the last two decades of his life was dealing with the applications of functional equations in the behavioral sciences, in collaboration with Aczél.

He also wrote then his last book:

- Luce, R.D. *Utility of Gains and Losses: Measurement-Theoretical and Experimental Approaches.* Mahwah, NJ: Erlbaum, 2000.
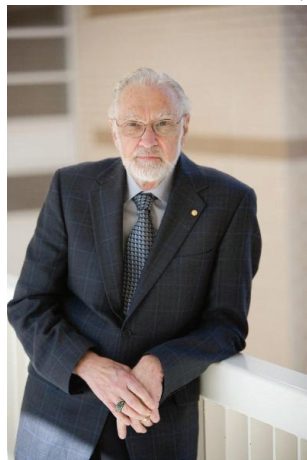
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It is also around that time that began his very fruitful collaboration with Ragnar Steimgrimson, which led them to write many joint papers.

In 2008, Ragnar Steingrimsson and Duncan Luce received the Outstanding Paper Award for 2005-07 from the Journal of Mathematical Psychology for their JMP paper, Vol. 51, pages 29-44.

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Through his scientific life, he wrote many papers in measurement theory with Louis Narens, who was a long time collaborator of Duncan.
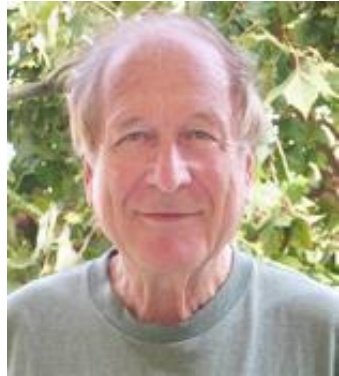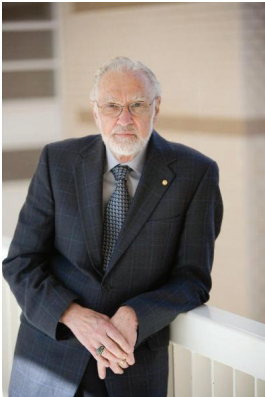
R. Duncan Luce, 1925-2012

One of the giants of mathematical psychology

We were all very lucky to know, and for some of us, to work with, not only a enormously productive, multifaceted scientist, but also a very kind and thoughtful human being.

R. Duncan Luce, 1925-2012
One of the giants of mathematical psychology

We will miss him.